THE NEMETH CODE
OF BRAILLE MATHEMATICS
AND SCIENTIFIC NOTATION
1965

AMERICAN PRINTING HOUSE FOR THE BLIND
Louisville, Kentucky
[illegible]

Spungin

HV1698
N343
1965.
c.4

THE NEMETH CODE
OF BRAILLE MATHEMATICS
AND SCIENTIFIC NOTATION
1965

Developed by Abraham Nemeth, Ph.D.
in Conjunction with the
Joint Committee on Mathematics and Scientific Notation
of the
AAIB-AAWB Braille Authority and National Braille Association, Inc.

Adopted, January 1, 1965
by the
AMERICAN ASSOCIATION OF INSTRUCTORS OF THE BLIND
and the
AMERICAN ASSOCIATION OF WORKERS FOR THE BLIND

AMERICAN PRINTING HOUSE FOR THE BLIND
Louisville, Kentucky
1966

AMERICAN FOUNDATION FOR THE BLIND
15 WEST 16th STREET
NEW YORK, NY 10011

Digitized by the Internet Archive
in 2015

https://archive.org/details/nemethcodeofbrai00deve

CONTENTS

FOREWORD ix

AUTHOR'S FOREWORD x

INTRODUCTION xii

RULE I. Braille Indicators 1
  1. Definition 1

RULE II. Numerals and the Numeric Indicator 1
  2. Representation of numerals 1
  3. Use of the numeric indicator 1
  4. Non-use of numeric indicator 3
  5. Effectiveness of the numeric indicator 7
  6. Long numerals 7
  7. Numerals with ordinal endings 7
  8. Plural of numerals 7
  9. Numerals in figures and diagrams 7
  10. Roman numerals 8

RULE III. Punctuation and Other Marks 8
  11. The punctuation indicator 10
  12. Use of the punctuation indicator 10
  13. Non-use of the punctuation indicator 10
  14. Effectiveness of the punctuation indicator 11
  15. Apostrophe-s combination 11
  16. Asterisk, Dagger 12
  17. Check mark, Ditto mark 12
  18. Comma 12
  19. Dash, long 13
  20. Ellipsis 13
  21. Exclamation point 13
  22. Hyphen 13
  23. Paragraph mark, Section mark 13
  24. Parentheses, Braces, Brackets 13
  25. Use of literary parentheses, braces and brackets 13
  26. Non-use of literary parentheses, braces and brackets 14
  27. Period 14
  28. Tally marks 14

RULE IV. Fractions and Fraction Indicators 15
  29. Simple fraction 16
  30. Use of simple-fraction indicators 16
  31. Non-use of simple-fraction indicators 17
  32. Mixed numbers 18

33. Complex fraction 18
34. Use of complex-fraction indicators 18
35. Non-use of complex-fraction indicators 18
36. Hypercomplex fraction 19
37. Use of hypercomplex-fraction indicators 19
38. Continued fraction 20
39. Use of spatial arrangements for fractions 20
40. Permissive use of spatial arrangement for fractions 22

RULE V. Omissions and the Omission Indicator 23
41. Use of omission indicator 23
42. Non-use of omission indicator 23
43. Punctuation of the omission indicator 23

RULE VI. Cancellation and Cancellation Indicators 24
44. Use of cancellation indicators 24

RULE VII. Capitalization, Alphabets, Type Forms, and Associated Indicators 26
45. Use of capitalization indicator 26
46. Non-use of the capitalization indicator 26
47. Effectiveness of the capitalization indicator 27
48. Alphabets 27
49. Alphabetic indicators 27
50. Effectiveness of alphabetic indicator 28
51. Use of Roman-letter indicator 29
52. Non-use of Roman-letter indicator 30
53. Type form of letters 31
54. Use of the type-form indicator 31
55. Non-use of type-form indicator 31
56. Effectiveness of type-form indicator 32
57. Type forms of numerals 32
58. Boldface type 32
59. Non-use of type-form indicators 33
60. Plural of letters 33
61. Punctuation of single letters 33
62. Letters with ordinal endings 34

RULE VIII. Subscripts, Superscripts, and Level Indicators 34
63. Nature of subscripts and superscripts 35
64. Order of subscripts and superscripts 35
65. Level indicators 35
66. Use of level indicators 35
67. Orientation by level indicator 35
68. Left subscripts and superscripts 38
69. Direct subscripts and superscripts 39
70. Numeric subscripts 39

71. Use of base-line indicator 40
72. Non-use of base-line indicator 41
73. Effectiveness of level indicator 41
74. Simultaneous subscripts and superscripts 43
75. Primes in addition to subscripts or superscripts 43
76. Plural of mathematical expressions which end with a subscript or superscript 44

RULE IX. Modifiers and Modification Indicators 44
77. Modifier 45
78. The five-step rule for transcribing modified expressions 45
79. Modifiers of higher order 46
80. Simultaneous modifiers 47
81. Horizontal bars, equals sign 47
82. Binomial coefficient 47
83. Modified expressions in subscripts and superscripts 48
84. Plural of modified expressions 48
85. Interior modifiers with signs of shape 48
86. Arc 48
87. Arrows 49
88. Horizontal bar 50
89. Vertical bar 51
90. Caret 51
91. Dot 52
92. Equilateral triangle 52
93. Hollow dot 53
94. Question mark 53
95. Tilde 53

RULE X. The Multipurpose Indicator 53
96. Use of the multipurpose indicator 53

RULE XI. Radicals and Radical Indicators 54
97. Six-step rule for transcribing radicals 54

RULE XII. The Enlargement Indicator and Systems of Mathematical Expressions 56
98. Use of enlargement indicator 57
99. Non-use of enlargement indicator 58

RULE XIII. Shapes and Shape Indicators 58
100. Shape 62
101. Other shapes 63
102. Shape with structural modification 64
103. Shape with interior modification 65
104. Shape modified by superposition 65
105. Corners 66
106. Plural of a sign of shape 67

RULE XIV. The Use of Contractions and Short-form Words 67
107. Use of contractions and short-form words 67
108. Non-use of contractions and short-form words 67

RULE XV. Signs of Operation 69
109. Ampersand 71
110. Asterisk, Paragraph mark, Section mark 71
111. Circle with interior dot 71
112. Circle with interior plus 71
113. Comma 71
114. Division 72
115. Filled-in dot 72
116. Filled-in square 72
117. Fraction lines 72
118. Hollow dot 72
119. Hollow square 72
120. Intersection, Union 72
121. Logical product, Logical sum 73
122. Minus 73
123. Minus followed by plus 73
124. Minus or plus 73
125. Multiplication 74
126. Number sign, Cross-hatch, Tic-Tac-Toe 74
127. Plus 74
128. Plus followed by minus 74
129. Plus or minus 74

RULE XVI. Comparison Signs 75
130. Negation signs 82
131. Simple comparison signs 83
132. Modified comparison signs 86
133. Comparison signs compounded vertically 86
134. Comparison signs compounded horizontally 87
135. Comparison signs compounded by superposition 87

RULE XVII. Representation of Arrows 87
136. Arrow components 89
137. Six-step rule for construction of arrows 89
138. Arrow directions 90
139. Arrow shafts 91
140. Arrow types 92
141. Arrowheads 92

RULE XVIII. Grouping Signs 94
142. Signs of grouping 95
143. Enlarged grouping signs 95

144. Angle brackets 95
145. Horizontal grouping signs 95
146. Boldface brackets 96
147. Mathematical grouping symbols 96
148. Use of mathematical grouping symbols 96
149. Vertical bars 97

RULE XIX. Other Signs 98
150. At 99
151. Caret 99
152. Cent, Dollar, Percent, Pound sterling 99
153. Crossed h 100
154. Decimal 100
155. Empty set, (null set, void set) 100
156. Existential quantifier 100
157. Factorial 100
158. Infinity 101
159. Integral 101
160. Prime 101
161. Round d, partial derivative 102
162. Since, therefore 102
163. Universal quantifier 102

RULE XX. Abbreviations 103
164. Abbreviations of measurement 104
165. Literary abbreviations 105
166. Abbreviated scientific words and phrases 106
167. Logarithms 106
168. Trigonometric and hyperbolic functions 106
169. Limit 106

RULE XXI. Signs and Symbols of Format 107
170. Division format 108
171. Separation line 110
172. Square root format 112
173. Synthetic division format 112
174. Special arrangement for multiplication 113

RULE XXII. Spacing 113
175. Spacing with symbols of operation 113
176. Spacing with symbols of comparison 115
177. Spacing with symbols of grouping 116
178. Spacing with symbols of shape 116
179. Spacing with abbreviations of measurement 117
180. Spacing with abbreviated scientific words 117

181. Argument 118
182. Spacing rule for arguments 119
183. Spacing with separation lines 120

RULE XXIII. Runover and Spatial Arrangements 120
184. Runovers of mathematical expressions 120
185. Spacing with spatial arrangement 123
186. Alignment of fractions 123

APPENDIX 125

INDEX OF SYMBOLS 130

GENERAL INDEX 164

FOREWORD

THE NEMETH CODE OF BRAILLE MATHEMATICS was officially adopted by the AAIB and AAWB in 1952. In response to the many requests for clarification, interpretation, and expansion, from students, transcribers and educators, and in answer to the objections raised by advocates of other braille mathematics codes, the AAIB-AAWB Braille Authority began its study of all existing braille mathematics codes in 1959. With the aid of an advisory committee of mathematicians, educators and transcribers, codes from England, France, Germany and Japan, as well as the old Taylor and 1952 Nemeth codes, were analyzed and tested.

These codes were examined for five basic qualities: (1) clarity of reading; (2) correlation between ink-print and braille representations; (3) ease of transcribing; (4) systematic progression of braille symbols; and (5) expandability for the future inclusion of new mathematical and scientific symbols.

This investigation led to the conclusion that the principles of the Nemeth Code showed the greatest potential for the incorporation of all these desired qualities. The modification and amplification of the 1952 code was then undertaken and has resulted in the development of THE NEMETH CODE OF BRAILLE MATHEMATICS AND SCIENTIFIC NOTATION, 1965.

Grateful acknowledgment for an outstanding contribution is accorded the National Braille Association, Inc., which has given both technical and financial support to this important project since 1962. Its Committee on Braille Mathematics and Scientific Notation has joined forces with the Advisory Committee on Mathematics and Scientific Notation of the Braille Authority to make a real contribution towards bringing the project to its present stage of development, and is continuing to serve in research looking to further refinements.

The Braille Authority is deeply indebted for the valuable contributions of services rendered by the following individuals:

Joint Committee on Mathematics and Scientific Notation

National Braille Association Representatives	Braille Authority Representatives
Vera P. Clark	Robert E. Bruce
Helen Epstein	Kenneth R. Ingham
Paul Humphrey	Paul C. Mitchell
George A. Locke	Earl Nickerson

Abraham Nemeth, Consultant and Developer

Credit and appreciation is also given to Services to the Blind, Vocational Rehabilitation Administration, Department of Health, Education, and Welfare, Washington, D. C., for a very substantial grant to the American Printing House for the Blind, Louisville, Kentucky, which is also subsidizing the publication of this Code. Through their generosity, the complete plate costs, plus 50 percent of the cost of the printing and binding of the first editions in both ink print and braille, have been underwritten, thereby making possible distribution of this work at a minimum price.

Upon adoption by the AAIB and AAWB, as of January 1, 1965, this work becomes the official codebook on mathematics and scientific notation for use in the United States.

AAIB-AAWB Braille Authority

Maxine B. Dorf (1959-)
Marjorie S. Hooper (1959-)
Bernard M. Krebs (1959-)

AUTHOR'S FOREWORD

This braille code for mathematical and scientific notation has been prepared to provide a system of braille symbols which will allow technical literature to be presented and read in braille, and to convey to the blind reader as vivid a picture of the signs used in print as is conveyed to the sighted reader.

It is appropriate, at this point, to make some remarks concerning the rationale and philosophy upon which the Code has been built.

Although the Code is intended to be as complete as possible, finality, unfortunately, can never be achieved by any code. In the rapid development in the fields of science and technology which is currently being experienced, new signs are constantly being devised and old ones modified. To meet this situation, there has been built into the Code a system of *braille indicators* designed to make the Code extendable so that braille symbols can be constructed for new signs, as they find their way into print, in a systematic manner.

The Code is intended to be as uniform as possible. To achieve this aim, the rules have been formulated in such a way that the number of exceptions is kept to a minimum.

The Code is intended to convey as accurate an impression as is possible to the braille reader of the corresponding printed text. This is, perhaps, one of its principal features. When the braille reader has a clear conception of the corresponding printed text, the area of communication between himself and his teacher, his colleagues, and his associates is greatly broadened. *A test of the accuracy with which the Code conveys information from the print to the braille text is to effect a transcription in the reverse direction. The amount of agreement between the original printed text and one transcribed from the braille is a measure of the Code's accuracy.*

The rules of the Code are intended to require a minimum of decision-making and of interpretation on the part of the transcriber, because he cannot be expected to have a knowledge of all the technical and scientific fields which he is called upon to transcribe. Accordingly, a careful distinction is made between the meaning which a printed sign has and the sign itself. In this Code the transcriber is required to represent only the sign and to be unconcerned with the meaning of that sign. Of course, the majority of signs, particularly at the lower levels of mathematics, are signs about which there is universal agreement as to their meaning, and these constitute the core about which has grown the modern system of mathematical notation. However, at the more advanced levels, technical writers have, with increasing frequency, been assigning new and unusual meanings to many of the signs which have long been "standard," in addition to inventing new signs. In view of this situation, the only safe course is to transcribe the signs themselves and not their meanings. Because of this feature, there will, hopefully, be attracted a large number of transcribers to the cause of technical and scientific transcription for which the need is ever increasing.

This Code may be presented either from the point of view of a person who uses it to effect a transcription, or from the point of view of a braille reader with material already transcribed. Although both points of view have been constantly kept in mind, emphasis has necessarily been placed on the former. The problem of transcription is intrinsically more difficult than the problem of reading because the transcriber must actually recall to mind the specific symbols which must be used and the rules which govern their use, whereas the braille reader must only recognize the symbols which he encounters and be only slightly aware of the underlying rules.

These remarks must not lead to the inference that the Code has been constructed with the needs of the transcriber superseding those of the braille reader. On the contrary, the symbols and rules of the Code have been devised with the needs of the braille reader as the chief consideration. For his benefit, there is included in this presentation an INDEX OF BRAILLE SYMBOLS the entries of which have been arranged in accordance with the standard arrangement of the 63 braille symbols.

In this presentation, each rule is introduced by a list of signs and symbols for quick reference when the rule is already known and only a specific representation is wanted. The body of the presentation is organized into sections which are consecutively numbered and captioned for easy reference, and contains the rules, explanations, and examples of the use of the Code. There follows an APPENDIX which lists combinations of type-form, alphabetic, and capitalization indicators, as well as the five alphabets for which specific provision has been made in this Code. For complete cross-referencing, there then follow the INDEX OF BRAILLE SYMBOLS and the GENERAL INDEX.

Throughout this presentation, the word *sign* is used in referring to a character or sequence of characters in print; the word *symbol* is used in referring to a character or sequence of characters in braille. Where possible, the name of the sign is furnished in the reference list. Sometimes the name of a sign derives from the mathematical meaning which it has. Simple examples are the *plus sign*, the *decimal point*, and the *percent sign*. In these cases the transcriber must be particularly alert because these signs must be transcribed by means of their corresponding braille symbols even when such a sign is used for an entirely different purpose. Other signs have names which are descriptive of the signs themselves, such as *arrow*, *vertical bar*, and *diagonal line*. Still others have names in accordance with the way in which they are read aloud, such as *is less than*, *is contained in*, and *is an element of*. Some signs have no name at all. In all these cases, the transcriber must not permit the name of the sign to be the determining factor in choosing the braille symbol to be used.

In most cases, examples are furnished with the individual items. Each example is intended to illustrate a different point. It is hoped that the examples are sufficiently suggestive so that they may be imitated with confidence in parallel situations. The parenthetical descriptions which accompany the examples are primarily for the guidance of the braille reader. In the print copy, the actual signs for showing the examples are presented in addition to these parenthetical descriptions.

The presentation which follows must be regarded as the basic Code. It is the intention to issue specialized applications in the form of supplements. At the present time, two such supplements are contemplated. The first is intended to deal with contracted representations and the purpose is to provide a means for expressing modifications and shape symbols in a fewer number of cells than is required in the basic Code. The second supplement is intended to be an application of the basic Code to the constructions in chemistry. The examples in this basic presentation are drawn principally from the central core of pure mathematics. Thus, physicists, statisticians, and logicians will find few examples in their fields. Nevertheless, the symbols, rules, and constructions of the basic Code apply with equal force to these fields. If it should become desirable to do so, additional supplements could be issued showing how the Nemeth Code is applied to constructions which are typical in these fields.

It is important that this presentation be accepted quite literally and that no meaning be imputed to the rules and principles which is not expressly stated or implied. It may sometimes appear quite fortuitous and arbitrary that a particular sign has been classified in a section which the reader's past experience or training indicates is inappropriate or even wrong. For the purposes of this Code, however, the transcriber must accept the classification as well as the rules herewith submitted, past experience or technical training notwithstanding.

The transcriber may feel, in certain situations, that some constructions are excessively long and may be tempted to shorten the construction by the use of a symbol of his own invention. *The transcriber is enjoined against yielding to this temptation.* The Code has been formulated in such a way that the same construction gives the same information to the braille reader from elementary arithmetic to the most advanced mathematics. Therefore, tampering with the constructions presented herein would have the effect of destroying this uniformity.

The author wishes to express his most particular appreciation to the National Braille Association, Inc. This organization has been involved in a major way in the use, extension, and clarification of the Nemeth Code since the appearance of the 1956 revision through its Committee on Mathematics and Scientific Notation. Without its moral, technical, and financial support, and without the collective experience in the use of the Code by dozens of its volunteer members, the present status of the Code with respect to completeness and organization could never have been attained. More specifically, the author wishes to express his appreciation to the following people:

Mrs. Alexander B. Clark whose masterful screening of information and questions from the field saved the author from being overwhelmed by the sheer bulk of his task.

Mr. George A. Locke for his painstaking and faithful transcription of several braille drafts of the Code into print, with all the attention to minute detail that is involved in such a task.

Mr. Paul Humphrey for arranging to make available the resources by which such printed materials could be duplicated and distributed.

Mrs. Daniel Epstein for her careful records on the basis of which revisions and corrections could be made precisely as agreed upon at the committee meetings.

Mr. Bernard M. Krebs and the AAIB-AAWB Braille Authority for promoting this joint undertaking which has culminated in the present publication.

Messrs. Kenneth R. Ingham, Earl Nickerson, Robert E. Bruce, and Paul C. Mitchell (deceased), as well as to all those named above, for critically examining the Code drafts, for their many helpful comments and suggestions, and for their assistance in guiding the organization of the completed work.

The author makes grateful acknowledgment to Mrs. Samuel Hartman of Detroit with whom he has had personal association for ten years, and who has transcribed for him more than 30,000 braille pages of mathematics at all levels during this period. The solution of many of the problems which she has encountered in such extensive use of the Code is reflected throughout this work.

For many, many days of painstaking and laborious proofreading through three drafts of the Code manual, and for preparation and typing of the indexes and prefatory materials, sincerest thanks are due to Mrs. Tom Downs, Mrs. Howard D. Kraus, Mrs. Harvey L. Leach, Mrs. Ruth Peters, and Mrs. Hyman Stein.

The author's fondest appreciation is reserved for his wife, Florence, whose constant encouragement and inexhaustible patience during the preparation of the Code manual, and whose graciousness to all the volunteers who came here to work on it, made the whole project go so smoothly.

Finally, the author wishes to thank all the students, teachers, and transcribers whose experiences in using the Code have pointed the way, directly or indirectly, toward its improvement, extension, and clarification.

Abraham Nemeth
Detroit, Michigan
1965

INTRODUCTION

Mathematical expressions are represented in print by the use of a large number of arbitrary signs. Among these signs there are the ten digits, the lower-case and capitalized letters of several alphabets, the script, italic, and boldface forms of these same letters, as well as numerous signs of operation, signs of comparison, signs of grouping, and many other signs serving the miscellaneous requirements of mathematical and scientific expression. Furthermore, mathematical signifiance is conveyed not only by those signs separately, but by their arrangement on levels above or below a reference line of writing, as well as by their arrangement above or below a simple or a complex fraction line.

With only 63 distinct braille characters available (64 if the space is counted), the accomplishment of this Code is to make provision for the representation of all of these signs, as well as to give an indication of their arrangement. It is, of course, impossible to establish a one-to-one correspondence between the 63 braille characters and the hundreds of signs used in modern mathematics. It is also impractical, as a general procedure, to imitate the arrangement of these signs at various levels relative to a reference line of writing or to a simple or complex fraction line.

Accordingly, the Code presented in the following pages is characterized by the use of a system of braille indicators. The *braille indicators* in this Code play the same role as do the composition signs of English Braille. In both systems, the braille indicators or the signs of composition have no meaning of their own, but have the power of imparting a meaning to the braille symbols which precede or follow them in accordance with well-formulated rules. While there are only six signs of composition in English Braille, there are more than 30 in this Code. It is by means of these indicators that the Code is extendable in a systematic manner so as to provide for new signs as they arise.

Among the functions of some of these indicators are the following: To distinguish between lower-case and capitalized letters within the same alphabet; to distinguish between the letters of one alphabet and those of another (five alphabets can be so distinguished at present); to distinguish between letters written in ordinary type and those written in italic type, boldface type, or script; to distinguish between small and large signs of grouping of the same type; to distinguish between signs at one level and those at another relative to a reference line of writing; to indicate the beginning and end of a simple or complex fraction; to indicate the termination of a radical. Braille indicators serve other purposes; the details are presented in the following pages.

The question naturally arises as to when to use the *Nemeth Code of Mathematics and Scientific Notation, 1965*. To answer this question, printed works are grouped into three principal categories:

Non-technical works are those which appear in prose, poetry, or similar style. The designation *non-technical* applies only with respect to mathematical and scientific notation. A work in law or medicine may be quite technical in those fields, but must be regarded as non-technical in the sense just mentioned. Of course, there will be no occasion to use the symbols or the rules of this Code in non-technical texts.

Partially technical works include science books written for the layman or textbooks in the social sciences which use mathematical terminology and notation. Such works are characterized by the use of an occasional mathematical sign or a small number of such signs. In works of this kind, the mathematical signs may be treated as in English Braille by replacing such signs by the corresponding words. This procedure is particularly suitable when there is no intention that the reader should manipulate such signs for the purpose of solving equations or performing computations. Sometimes, however, the replacement of a sign by a corresponding word is not practical, especially when an aggregate of such signs appears in an arrangement which is unusual from the literary point of view. In such cases the transcriber should use the symbols and the rules of this Code with a note to the braille reader that this is being done.

Technical works are those in the fields of mathematics, including books in elementary arithmetic, physics, chemistry, or any of the natural sciences. Books in the social sciences which make a strong appeal to the use of mathematical signs and modes of expression must also be regarded as technical. For such works, the symbols and the rules of this Code must be used throughout. The transcriber must indicate at the beginning of each volume, by means of a transcriber's note, that the work has been transcribed in English Braille and in the Nemeth Code, particularly if the work is in one of the *social* sciences and it cannot be ascertained from its title whether it is technical or partially technical. This having been done, the transcriber is at liberty to make the transition from literary to scientific material and vice versa without any special indication to the braille reader. The reader, on the other hand, will be alerted to the technical nature of the work and will expect such transition to occur without any warning other than that which the context provides. It is assumed, of course, that the rules of English Braille and of the Nemeth Code will be rigidly followed within their respective domains.

As has been indicated above, the context almost always is of such a nature that the braille reader expects a transition from one code to the other at the precise point at which it occurs. Furthermore, the misinterpretation of mathematical symbols as being literary or vice versa almost always leads to an unmeaningful combination. Because of these two factors, no confusion will occur in the mind of the braille reader as to which code is being used at any particular time.

Signs which for many decades have been exclusively associated with college and graduate mathematics have in recent years been filtering down to high school and grade school levels. Thus, the set operations of *union*, *intersection*, and *inclusion*, which were traditionally encountered for the first time by a mathematics major in his junior year at college, have now become fairly commonplace at the fourth or fifth grade level, and are first met even earlier. In addition, grade school and high school mathematics is now being presented with considerably more rigor than heretofore, and nuances in meaning are being preserved and even emphasized by the use of distinct signs having similar, but not equivalent, meanings. In keeping with this spirit, the Code furnishes distinct braille symbols corresponding to distinct signs in print. In particular, at the lower levels of mathematics, this Code maintains a distinction between the horizontal and diagonal fraction lines, and between the *dot* and the *cross* which signify multiplication.

Constructive and critical comments are invited. They should be sent to the Advisory Committee on Mathematics and Scientific Notation of the Braille Authority.

THE NEMETH CODE OF BRAILLE MATHEMATICS AND SCIENTIFIC NOTATION

1965

RULES

RULE I—BRAILLE INDICATORS

§1. **Definition:** A *braille indicator* is a symbol for which, with the exception of the cancellation indicators, there is no equivalent sign of print.

The various braille indicators are shown with their associated rules. For the various indicators which are used in this Code, see Rules II-XIII.

The use of braille indicators in this Code constitutes an extension of the principles which underlie the composition signs of English Braille. By their use, it is possible to construct the hundreds of symbols required in modern mathematical and scientific expression. By their use, it is also possible to convey the "two-dimensional" information contained in ink print through the medium of the Braille System whose nature is essentially "one-dimensional."

RULE II—NUMERALS AND THE NUMERIC INDICATOR

Numeric Indicator ⠼ (dots 3-4-5-6)

Numerals

⠼⠂	1	⠼⠆	2	⠼⠒	3	⠼⠲	4	⠼⠢	5
⠼⠖	6	⠼⠶	7	⠼⠦	8	⠼⠔	9	⠼⠴	0

§2. **Representation of Numerals:** The numerals 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0 are represented, respectively, by the symbols whose configurations correspond to the symbols for the letters a to j, but which occupy the lower portion of the braille cell. However, English Braille numerals must be used for page numbers or volume numbers, or references to them. In the above list, each numeral is preceded by the numeric indicator. The numeric indicator is not required with each numeric symbol; however, when it *is* used, it identifies the symbols which follow it as numerals.

§3. **Use of the Numeric Indicator:** The numeric indicator must be used:

a. As the first symbol on the braille line and before a numeric symbol or decimal point; that is, the first symbol of a braille line must not be numeric.

(1) ⠼⠆⠶ 27

(twenty-seven at beginning of braille line)

b. Except as in §4, after a space and before a numeric symbol or decimal point.

(1) ⠠⠐⠮ ⠶ ⠼⠶ ⠃⠁⠇⠇⠎⠲ There were 7 balls.

(2) ⠼⠂⠬⠭⠬⠽ ⠨⠅ ⠼⠴ $1 + x + y = 0$

(1 plus x plus y equals 0)

(3) ⠼⠨⠆ ⠨⠂ ⠼⠨⠂ $.2 > .1$

(two-tenths is greater than one-tenth)

(4) ⠽ ⠨⠅ ⠼⠆⠎⠊⠝ ⠭ $y = 2 \sin x$

(y equals 2 sine x)

c. After a symbol of punctuation other than the hyphen or mathematical comma in numbers and before a numeric symbol or decimal point.

(1) ⠦⠼⠒ ⠙⠕⠛⠎⠴ "3 dogs"

(left outer quote, 3 dogs, right outer quote)

(2) ⠠⠏⠗⠕⠃⠁⠃⠊⠇⠰⠽⠤⠤⠼⠴ Probability—0

(zero after the dash)

(3) ⠼⠒⠸⠒⠼⠒⠴⠤⠲⠸⠒⠼⠲⠢ 3:30-4:45

(three thirty to four forty-five)

(4) ⠦⠼⠨⠢ ".5

(left outer quote followed by five-tenths)

(5) ⠼⠖⠢⠤⠶⠢ 65-75

(65 to 75)

(6) ⠼⠂⠴⠠⠴⠴⠴ 10,000

(comma after the first zero)

d. Between a boldface, italic, or script indicator and a numeric symbol or decimal point. (See also §57; see §58**a** for exception.)

(1) ⠨⠼⠒ *3*

(italicized 3)

(2) ⠸⠼⠴ **0**

(a boldface zero)

(3) ⠨⠼⠨⠒ *.3*

(three-tenths, italicized)

(4) ⠈⠼⠆ *2*

(script 2)

(See also §57; see §58a for exception.)

e. After the symbol for *minus* and before a numeric symbol or decimal point, provided that the minus symbol is preceded by a space, a symbol of punctuation, or that it occurs at the beginning of a braille line.

(1) ⠤⠼⠂ —1

(minus 1 after space or at beginning of braille line)

(2) ⠐⠤⠶ —7

(minus 7 at beginning of braille line, run over from previous line where superscript indicator was in effect; numeric indicator is not required because the symbol for *minus* is not the first symbol of the braille line.)

(3) ⠦⠤⠼⠲ "—4

(—4 after the symbol for the left outer quotation mark)

§4. Non-use of Numeric Indicator: The numeric indicator must not be used:

a. After a space and before a numeric symbol or decimal point if the corresponding sign follows an abbreviated scientific word or phrase (see list at beginning of Rule XX). If such an abbreviated word or phrase carries a superscript or subscript, these must not become the reason for using the numeric indicator.

(1) ⠎⠊⠝ ⠆⠭ sin 2x

(sine of 2 x)

(2) ⠎⠊⠝⠘⠆ ⠆⠭ $\sin^2 2x$

(sine squared of 2 x)

(3) ⠇⠕⠛⠂⠴ ⠆ $\log_{10} 2$

(logarithm to the base 10 of 2)

b. After a space and before a numeric symbol or decimal point when the numerals are enclosed within parentheses, brackets, or other symbols of grouping, *and are separated by commas*, or when they are entries in a determinant or matrix. When a list of numerals so enclosed is so long as to require a runover, the numeric indicator must not be used before the first numeric symbol on the new line. However, when numerals which are separated by commas are not enclosed, the numeric indicator must be used.

(1) ⠷⠴⠠ ⠴⠾ (0, 0)

(the point with coordinates 0, 0)

(2) ⠷⠂⠬⠓⠠⠀⠆⠬⠅⠠⠀⠴⠾ (1 + h, 2 + k, 0)

(the point with coordinates 1 + h, 2 + k, 0)

(3) ⠷⠴⠠⠀⠤⠂⠠⠀⠬⠤⠆⠾ (0, — 1, ± 2)

(the point with coordinates 0, minus 1, plus or minus 2)

(4) ⠈⠷⠴⠠⠀⠂⠈⠾ [0, 1]

(the closed interval from 0 to 1; the number pair is enclosed within brackets)

(5) ⠷⠭⠀⠨⠅⠀⠼⠴⠾ (x = 0)

(x equals 0; although the mathematical expression is enclosed, the configuration is not one in which the numeric symbols are separated by commas; the numeric indicator is therefore required)

(6) ⠠⠮⠀⠔⠞⠑⠛⠻⠎⠀⠼⠆⠠⠀⠼⠲⠠⠀⠼⠖⠀⠜⠑⠀⠑⠧⠢⠲

The integers 2, 4, 6 are even.

(Since the sequence of numerals is not enclosed, the numeric indicators are required.)

(7) ⠠⠳⠂⠀⠆⠠⠳
⠠⠳⠒⠀⠲⠠⠳

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

(the determinant with entries 1, 2, 3, 4)

(8) ⠷⠂⠠⠀⠆⠠⠀⠒⠠⠀⠲⠠
⠢⠠⠀⠖⠠⠀⠶⠾

(1, 2, 3, 4,
5, 6, 7)

(numeric indicator not used on line where runover occurs)

c. After a space and before a numeric symbol if the space *follows* the symbol for the ellipsis.

(1) ⠼⠴⠨⠒⠒⠒⠀⠄⠄⠄⠀⠒⠀⠄⠄⠄ 0.333 . . . 3 . . .

(the 3 which occurs between the two ellipses does not require the numeric indicator)

(2) ⠼⠢⠠⠀⠼⠂⠴⠠⠀⠼⠂⠢⠠⠀⠄⠄⠄⠠⠀⠼⠂⠴⠴ 5, 10, 15, . . . , 100

(the space follows a comma, and not the ellipsis; therefore the numeric indicator is required before the 100)

d. After a space and before a numeric symbol or decimal point when numerals are arranged for the purpose of computation or are displayed in the columns of a table or chart. However, the numeric indicator is required after a space and before a numeric symbol or decimal point in a fraction arranged spatially.

(1)
⠂⠦⠀⠗⠐⠆
⠒⠒⠒⠒⠒⠒⠒⠒⠒
⠆⠢⠕⠲⠢⠆

$$\begin{array}{r} 18\ \ r2 \\ 25\overline{)452} \end{array}$$

(452 divided by 25 yields a quotient of 18 and a remainder of 2)

(2)

⠀⠀⠆⠶⠒
⠀⠬⠀⠦⠢
⠒⠒⠒⠒⠒⠒

$$\begin{array}{r}273\\+85\\\hline\end{array}$$

(273 plus 85)

(3)

⠀⠀⠲⠆⠖
⠀⠈⠡⠒⠲
⠒⠒⠒⠒⠒⠒

$$\begin{array}{r}426\\\times 34\\\hline\end{array}$$

(426 multiplied by 34)

(4)

⠀⠼⠂⠂
⠹⠒⠒⠒⠼
⠀⠀⠼⠢

$$\frac{11}{5}$$

(the fraction 11 over 5)

(5)

⠀⠀⠀⠼⠂⠬⠒
⠀⠀⠹⠒⠒⠒⠒⠼
⠀⠀⠀⠼⠲⠬⠢
⠠⠹⠒⠒⠒⠒⠒⠒⠠⠼
⠀⠀⠀⠼⠒⠬⠲
⠀⠀⠹⠒⠒⠒⠒⠼
⠀⠀⠀⠼⠢⠬⠖

$$\frac{\frac{1+3}{4+5}}{\frac{3+4}{5+6}}$$

(the complex fraction whose numerator is 1 + 3 over 4 + 5, and whose denominator is 3 + 4 over 5 + 6)

(6)

⠀⠙⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠼⠆⠭
⠹⠒⠒⠼⠞⠁⠝⠘⠤⠂⠀⠹⠒⠒⠒⠒⠼
⠀⠙⠭⠀⠀⠀⠀⠀⠀⠀⠀⠀⠼⠂⠤⠭

⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠙⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠙
⠀⠀⠀⠀⠀⠀⠀⠀⠷⠂⠤⠭⠾⠹⠒⠒⠼⠷⠆⠭⠾⠤⠆⠭⠹⠒⠒⠼⠷⠂⠤⠭⠾
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠙⠭⠀⠀⠀⠀⠀⠀⠀⠀⠀⠙⠭
⠀⠀⠀⠀⠀⠀⠠⠹⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠠⠼
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠷⠂⠤⠭⠾⠘⠆
⠨⠅⠀⠠⠠⠹⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠠⠠⠼
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠼⠆⠭
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠼⠂⠬⠷⠹⠒⠒⠒⠒⠼⠾⠘⠆
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠼⠂⠤⠭

$$\frac{d}{dx}\tan^{-1}\frac{2x}{1-x}$$

$$= \frac{\dfrac{(1-x)\dfrac{d}{dx}(2x)-2x\dfrac{d}{dx}(1-x)}{(1-x)^{2}}}{1+\left(\dfrac{2x}{1-x}\right)^{2}}$$

e. In any situation which has not been explicitly indicated.

(1) ⠼⠒⠖⠬⠲⠬⠢ ⠨⠅ ⠼⠲⠢ $36+4+9=49$

(2) ⠐⠶⠣⠱⠻ $\overline{7}$

(the numeral 7 is modified by a bar directly over it)

(3) ⠘⠒⠐⠭ ${}^{3}x$

(3 is a left superscript to x)

§5. **Effectiveness of the Numeric Indicator:** For the reader, the effectiveness of the numeric indicator is *continued* by a numeric symbol, decimal point, hyphen, or comma; its effectiveness is *terminated* by a space, by transition to another braille line, or by any braille symbol other than a numeric symbol, decimal point, hyphen or comma.

(1) ⠼⠒⠴⠨⠢⠶ 30.57

(30, decimal point, 57)

(2) ⠼⠂⠠⠴⠴⠴⠠⠴⠴⠴ 1,000,000

(one million)

§6. **Long Numerals:** Long numerals that cannot be accommodated on the braille line may be divided and run over to another line. Such a division may only be made after a comma, if present, and a hyphen must be supplied. The numeric indicator must be used as the first braille symbol of the braille line to which the numeral has been run over.

(1) ⠼⠂⠴⠴⠠⠴⠴⠴⠠⠴⠴⠴⠠⠤ 100,000,000,
⠼⠴⠴⠴ 000

(one hundred billion)

§7. **Numerals with Ordinal Endings:** When ordinal endings are required, they must be adjoined directly to the numeral by using the letters of the ordinal endings. The contractions for *st* and *th* must not be used. Ordinal endings are part of a mathematical expression and must be punctuated accordingly. (For letters with ordinal endings, see §62.)

(1) ⠼⠂⠎⠞⠠ ⠼⠆⠝⠙⠠ ⠼⠒⠗⠙⠠ ⠯ ⠼⠲⠞⠓⠸⠲

1st, 2nd, 3rd, and 4th.

(first, second, third, and fourth)

§8. **Plural of Numerals:** (See §15.)

(1) ⠠⠉⠨⠞ ⠃⠽ ⠼⠂⠴⠸⠄⠎⠸⠲ Count by 10's.

(punctuation indicator is required before the period)

(2) ⠠⠥⠎⠑ ⠕⠝⠇⠽ ⠼⠴⠸⠄⠎ ⠯ ⠼⠂⠸⠄⠎⠤⠤
⠥⠎⠑ ⠮ ⠃⠔⠜⠽ ⠎⠽⠎⠑⠍⠲

Use only 0's and 1's—use the binary system.

(punctuation indicator is not required before the dash)

(3) ⠠⠁ ⠼⠖⠠ ⠞⠺⠕ ⠼⠶⠸⠄⠎⠠ ⠯ ⠁⠝ ⠼⠦⠸⠲

A 6, two 7's, and an 8.

(punctuation indicator is not required before the second comma)

§9. **Numerals in Figures and Diagrams:** In figures and diagrams which contain numeric labels, the numeric indicator should, if possible, be used. The space to accommodate the numeric indicator may often be gained by a sufficient enlargement of the figure or diagram in question.

§10. Roman Numerals:

a. Except as in b below, Roman numerals must be transcribed as in English Braille. The Roman-letter indicator must be used before a Roman numeral if its letters are in lowercase; it must not be used before capitalized Roman numerals. The single capital symbol must be used before a Roman numeral consisting of a single capitalized letter; the double capital symbol must be used before a Roman numeral consisting of more than one capitalized letter.

(1) ⠰⠊⠠ ⠰⠊⠊⠠ ⠰⠊⠊⠊⠠ ⠰⠊⠧⠠ ⠰⠧⠸⠲ i, ii, iii, iv, v.

(the Roman numerals from 1 to 5 in lower-case letters)

(2) ⠠⠊⠠ ⠠⠠⠊⠊⠠ ⠠⠠⠊⠊⠊⠠ ⠠⠠⠊⠧⠠ ⠠⠧⠸⠲

I, II, III, IV, V.

(the Roman numerals from 1 to 5 in capitalized letters)

b. When Roman numerals are used within formulas or for performing computation, or when they carry primes, subscripts, superscripts, or modifiers, they must be transcribed as if they were composed of individual letters and without regard to their nature as Roman numerals. This means that the Roman-letter indicator must not be used before lower-case Roman numerals, and that the letters of a capitalized Roman numeral must be individually capitalized.

(1) ⠠⠧⠠⠊⠠⠊⠬⠠⠧ ⠨⠅ ⠠⠭⠠⠊⠠⠊ VII + V = XII

(capitalized Roman numeral 7 plus capitalized Roman numeral 5 equals capitalized Roman numeral 12)

(2) ⠠⠊⠠⠊⠄ II′

(the Roman numeral two prime)

(3) ⠐⠠⠍⠣⠱⠻ $\overline{\text{M}}$

(bar over M; the Roman numeral 1,000,000)

c. When a letter sequence does not represent a Roman numeral, or when it is doubtful that it does, the letters in the sequence must be individually capitalized if they are capitalized in print.

(1) ⠠⠉⠠⠇ CL

(the capital C followed by capital L may or may not be a Roman numeral)

d. Even when Roman numerals are transcribed as in English Braille, they must be regarded as mathematical expressions and punctuated accordingly (see examples (1) and (2) of a above, with particular attention to the punctuation therein).

RULE III—PUNCTUATION AND OTHER MARKS

Punctuation Indicator ⠸ (dots 4-5-6)

Punctuation Marks

Apostrophe ’ ⠄ (dot 3)

Asterisk * ⠈⠼ (dots 4, 3-4-5-6)

Braces			
Left	{	⠰⠶	(dots 5-6, 2-3-5-6)
Right	}	⠶⠆	(dots 2-3-5-6, 2-3)
Brackets			
Left	[	⠠⠶	(dots 6, 2-3-5-6)
Right	]	⠶⠄	(dots 2-3-5-6, 3)
Check Mark	✓	⠈⠜	(dots 4, 3-4-5)
Colon	:	⠒	(dots 2-5)
Comma	,	⠠	(dot 6)
Dagger	†	⠨⠳	(dots 4-6, 1-2-5-6)
Dash			
Long	——	⠤⠤⠤⠤	(dots 3-6, 3-6, 3-6, 3-6)
Short	—	⠤⠤	(dots 3-6, 3-6)
Ditto Marks	"	⠠⠄	(dots 6, 3)
Ellipsis	. . .	⠠⠠⠠	(dots 6, 6, 6)
Exclamation Point	!	⠖	(dots 2-3-5)
Hyphen	-	⠤	(dots 3-6)
Paragraph Mark	¶	⠈⠠⠏	(dots 4, 6, 1-2-3-4)
Parentheses			
Left	(	⠶	(dots 2-3-5-6)
Right	)	⠶	(dots 2-3-5-6)
Period	.	⠲	(dots 2-5-6)
Question Mark	?	⠦	(dots 2-3-6)

Quotation Marks			
Left Inner	‘	⠠⠦	(dots 6, 2-3-6)
Left Outer	“	⠦	(dots 2-3-6)
Right Inner	’	⠴⠄	(dots 3-5-6, 3)
Right Outer	”	⠴	(dots 3-5-6)
Section Mark	§	⠈⠠⠎	(dots 4, 6, 2-3-4)
Semicolon	;	⠆	(dots 2-3)
Tally	|	⠸	(dots 4-5-6)

§11. **The Punctuation Indicator** is not required with each symbol of punctuation; however, when it *is* used, it identifies the symbols which follow it as punctuation marks. (See §12-14.)

§12. **Use of the Punctuation Indicator:** The punctuation indicator must be used, except as in §13, after a mathematical symbol and before a punctuation symbol.

(1) ⠼⠴⠸⠲ 0.
(0 followed by period)

(2) ⠼⠴⠸⠄⠎ 0's
(0 followed by apostrophe-s)

(3) ⠠⠶⠍⠂⠝⠎ ⠼⠴⠸⠶⠄ [means 0]
(0 followed by right bracket)

(4) ⠰⠏⠸⠲ ⠼⠃⠛⠸⠲ p. 27.
(page twenty-seven)

§13. **Non-use of the Punctuation Indicator:** The punctuation indicator must not be used:

a. After a mathematical symbol and before a comma, hyphen, or dash.

(1) ⠼⠴⠠ 0,
(0 followed by comma)

(2) ⠼⠂⠴⠠⠴⠴⠴ 10,000
(comma after first 0)

(3) ⠼⠢⠤⠉⠢⠞ ⠌⠁⠍⠏ 5-cent stamp
(hyphen between 5 and cent)

(4) ⠼⠖⠢⠤⠤⠼⠶⠢ 65—75

(dash between 65 and 75)

b. After a space, another punctuation symbol, or at the beginning of a braille line.

(1) ⠦⠼⠴ “0

(0 preceded by left outer quotation mark)

(2) ⠦⠼⠴⠸⠴ “0”

(0 enclosed by outer quotation marks)

(3) ⠈⠼⠼⠂ *1

(asterisk followed by 1; the numeric indicator is required according to §3**c**, see also §16)

(4) ⠨⠳⠼⠂ †1

(dagger followed by 1; numeric indicator is required according to §3c)

(5) ⠈⠠⠎⠼⠆⠠ ⠈⠠⠏⠼⠒ §2, ¶3

(section 2, paragraph 3; numeric indicator is required according to §3**c**)

§14. **Effectiveness of the Punctuation Indicator:** The effectiveness of the punctuation indicator is continued by symbols of punctuation; its effectiveness is terminated by a space, by transition to another braille line, or by any braille symbol other than a symbol of punctuation.

(1) ⠼⠴⠸⠲⠴ 0.”

(0 followed by a period and a right outer quotation mark)

(2) ⠼⠴⠸⠄⠎⠸⠲ 0’s.

(0 followed by apostrophe-s followed by period; the second punctuation indicator is required because the effectiveness of the first is terminated by the letter s)

§15. **Apostrophe-s Combination:** The apostrophe-s combination may be adjoined to numerals, letters, and other mathematical expressions to form their plurals. When so adjoined, the apostrophe-s combination is part of a mathematical expression and must be punctuated accordingly.

(1) ⠼⠂⠸⠄⠎⠠ ⠼⠆⠸⠄⠎⠠ ⠯ ⠼⠒⠸⠄⠎⠸⠲ 1’s, 2’s, and 3’s.

(2) ⠰⠭⠸⠄⠎ ⠯ ⠰⠽⠸⠄⠎⠸⠲ x’s and y’s.

(3) ⠠⠉⠕⠇⠇⠑⠉⠞ ⠮ ⠭⠘⠆⠸⠄⠎ Collect the x^2’s

(collect the x squares)

(4) ⠠⠮ ⠉⠂⠸⠄⠎⠠ ⠉⠆⠸⠄⠎⠠ ⠠⠠⠠⠠ ⠉⠰⠝⠸⠄⠎⠸⠲

The c_1’s, c_2’s, . . . , c_n’s.

(the plurals of c_1, c_2, . . . , c_n)

(5) ⠫⠪⠸⠄⠎ ⠼⠂ ⠯ ⠼⠆ ∠s 1 and 2

(angles 1 and 2)

(6) ⠠⠮ ⠠⠕⠠⠓⠸⠄⠎⠸⠲ The OH's.

§16. **Asterisk, Dagger:** The asterisk and dagger must be transcribed as shown in the list of punctuation marks above, even when they are used for making *references;* the English Braille symbols must never be used. When it is certain that an asterisk or other sign is used for reference purposes, the superscript position should be ignored in the braille transcription.

(1) ⠈⠼⠼⠂ *1

(reference to note 1 by means of an asterisk; asterisk shown on base line in braille, even if it is in superscript position in print)

(2) ⠨⠳⠼⠂ †1

(reference to note 1 by means of a dagger; dagger shown on base line in braille, even if it is in superscript position in print)

§17. **Check Mark, Ditto Mark:** The check mark and the ditto marks must be separated from any other braille symbol by at least one space.

(1) ⠈⠜ ⠍⠊⠇⠅ √ milk

⠈⠜ ⠑⠶⠎ √ eggs

⠈⠜ ⠃⠗⠂⠙ √ bread

(a checked-off shopping list)

(2) ⠼⠆ ⠛⠕⠑⠎ ⠔⠞⠕ ⠼⠆⠠ ⠼⠂ 2 goes into 2, 1

⠠⠄ ⠠⠄ ⠠⠄ ⠼⠲⠠ ⠼⠆ " " " 4, 2

⠠⠄ ⠠⠄ ⠠⠄ ⠼⠖⠠ ⠼⠒ " " " 6, 3

(a partial table)

§18. **Comma:** The comma shown in the list of punctuation marks must be used only with mathematical expressions and is, for this reason, frequently referred to as the *mathematical comma.* The comma which follows a word must be transcribed as in English Braille and is called the *literary comma.* The comma at the end of a sequence of punctuation symbols which has been introduced by the punctuation indicator must be transcribed as the mathematical comma. The mathematical comma must be followed by a space, except when it separates parts of numbers, or is used as in §113, whether the space is present in print or not. (See §4**b** where some of the examples in print show no space after the comma.)

(1) ⠦⠼⠒⠸⠴⠠ "3",

(3 enclosed within outer quotation marks, followed by a comma)

§19. Dash, Long: The long dash must be used to indicate the omission of a word or phrase which the reader is *explicitly* requested to fill in (see §42b). The long dash is literary and must be punctuated as in English Braille.

(1) ⠤⠤⠤⠤⠂ ——,

(long dash followed by a comma)

(2) ⠤⠤⠤⠤⠲ ——.

(long dash followed by a period)

§20. Ellipsis: The ellipsis shown in the list of punctuation marks must be used only with mathematical expressions and is, for this reason, frequently referred to as the *mathematical ellipsis.* The ellipsis which occurs within a literary sentence must be transcribed as in English Braille and is the *literary ellipsis.* The ellipsis shown in the list of punctuation marks is a mathematical expression and must be punctuated accordingly.

(1) ⠠⠠⠠⠠ . . . ,

(mathematical ellipsis followed by mathematical comma)

(2) ⠠⠠⠠⠸⠲ . . . .

(mathematical ellipsis followed by period)

§21. Exclamation Point: The exclamation point is represented by the same sign of print as the factorial sign, and the judgment of the transcriber is required to choose between the corresponding symbols. The context is usually so clear in regard to this distinction that the possibility of doubt in choosing the proper symbol is remote.

§22. Hyphen: The hyphen is represented by the same sign of print as the minus sign. Since the corresponding braille symbols also coincide, no decision-making in this regard is required of the transcriber.

§23. Paragraph Mark, Section Mark: The paragraph and section marks must be transcribed as shown in the list of punctuation marks above, even when they are used to designate sections or paragraphs by number or letter; the English Braille symbols must never be used.

(1) ⠈⠠⠎⠼⠂⠠ ⠈⠠⠏⠁ §1, ¶a

(section 1, paragraph a)

§24. Parentheses, Braces, Brackets: The parentheses, braces, and brackets shown in the list of punctuation marks are those of English Braille and are frequently referred to as *literary parentheses, literary braces,* or *literary brackets.* By contrast, the parentheses, braces and brackets of §147 are used for grouping mathematical expressions and are frequently referred to as *mathematical parentheses, mathematical braces,* or *mathematical brackets.*

§25. Use of Literary Parentheses, Braces, and Brackets: Literary parentheses, braces and brackets must be used to enclose an expression, part of which is mathematical, but which begins with a word.

(1) ⠶⠠⠮⠕⠗⠑⠍ ⠼⠒⠲⠸⠶⠲ (Theorem 34).

(parenthetical reference to Theorem 34)

§26. **Non-use of Literary Parentheses, Braces, and Brackets:** Literary parentheses, braces, or brackets must not be used:

a. To enclose numerals or single letters. Such numerals or letters may either be within the body of the text or they may be used to identify formulas, the steps of an outline, or similar items. Mathematical parentheses, braces, or brackets must be used instead for these purposes (see §148a).

(1) ⠠⠋ ⠿⠍⠥⠇⠁⠎ ⠷⠂⠾ ⠯ ⠷⠆⠾ ⠺⠑
⠕⠃⠞⠁⠔ ⠮ ⠙⠑⠎⠊⠗⠫ ⠗⠑⠎⠥⠇⠞⠲

From formulas (1) and (2) we obtain the desired result.

(formula numbers are enclosed within mathematical parentheses)

b. To enclose an expression, part of which is mathematical, but which begins with a numeral or a mathematical expression.

(1) ⠷⠆⠠⠆⠲⠴ ⠏⠕⠥⠝⠙⠎⠾ (2,240 pounds)

§27. **Period:** The period must be omitted from abbreviations of measurement except when the period ends a sentence.

(1) ⠼⠖⠴ ⠍⠏⠓ 60 m.p.h.

(60 miles per hour; periods follow the lower-case letters in print)

(2) ⠼⠆⠢ ⠠⠎⠟ ⠠⠋⠞ 25 Sq. Ft.

(25 square feet; periods follow the q and the t in print)

(3) ⠼⠒⠋⠞⠐⠖⠊⠝ 3ft.6in.

(3 feet, 6 inches, written unspaced in print with periods following the abbreviations)

(4) ⠼⠒⠆ ⠋⠞⠸⠌⠎⠑⠉⠘⠆⠸⠲ 32 ft./sec.^{2}

(32 feet per second squared; period follows t, and exponent is to the right of the period in print; abbreviation ends the sentence)

§28. **Tally Marks:** Tally marks must be written consecutively up to a maximum of five tally marks. There must be exactly five tally marks in each group but the last. Groups of tally marks must be separated by a single space from each other and from surrounding material, but the transition to another line of braille takes the place of this required space. Transition to another line of braille must never be made from one tally mark to another within the same group.

(1) ⠸⠸⠸ |||

(three tally marks)

(2) ⠸⠸⠸⠸⠸ ⠸⠸⠸⠸ 𝍸 ||||

(nine tally marks)

(3) ⠸⠸⠸⠸⠸ ⠸⠸⠸⠸⠸ ⠸⠸⠸⠸⠸ ⠸⠸⠸⠸⠸ 𝍸 𝍸 𝍸 𝍸 ||

⠸⠸

(twenty-two tally marks, the last two on another braille line)

RULE IV—FRACTIONS AND FRACTION INDICATORS

Fraction Indicators

Complex

Opening ⠠⠹ (dots 6, 1-4-5-6)

Closing ⠠⠼ (dots 6, 3-4-5-6)

Fractional Part of a Mixed Number

Opening ⠸⠹ (dots 4-5-6, 1-4-5-6)

Closing ⠸⠼ (dots 4-5-6, 3-4-5-6)

Hypercomplex

Opening ⠠⠠⠹ (dots 6, 6, 1-4-5-6)

Closing ⠠⠠⠼ (dots 6, 6, 3-4-5-6)

Simple

Opening ⠹ (dots 1-4-5-6)

Closing ⠼ (dots 3-4-5-6)

Fraction Lines

Diagonal line / ⠸⠌ (dots 4-5-6, 3-4)

Horizontal line for ——

Simple Fraction ⠌ (dots 3-4)

Complex Fraction ⠠⠌ (dots 6, 3-4)

Spatial Arrangement ⠒⠒⠒⠒⠒ (dots 2-5, 2-5, 2-5, etc.)

§29. **Simple Fraction:** A *simple fraction* is one whose numerator and denominator contain no fractions except possibly at the subscript or superscript level.

§30. **Use of Simple-fraction Indicators:** Simple-fraction indicators must be used:

a. Except as in §31 below, to enclose a simple fraction whose numerator and denominator are separated by a horizontal fraction line in print.

(1) ⠹⠂⠌⠒⠼ $\frac{1}{3}$

(one third, horizontal line in print)

(2) ⠭⠘⠹⠂⠌⠆⠼ $x^{\frac{1}{2}}$

(x to the one-half power, horizontal line in print)

(3) ⠹⠁⠬⠃⠌⠉⠼ $\frac{a + b}{c}$

(a plus b, over c, horizontal line in print)

(4) ⠹⠭⠘⠹⠂⠌⠆⠼⠐⠌⠆⠼ $\frac{x^{\frac{1}{2}}}{2}$

(x to the one-half power, over 2; both fraction lines are horizontal in print, the expression is a simple fraction since the only fraction in its numerator is at the superscript level)

b. To enclose a simple fraction whose numerator and denominator are separated by a diagonal line in print, provided that the expressions on either side of the diagonal line appear at different levels relative to it, or in different type size than the surrounding text.

(1) ⠹⠁⠬⠃⠸⠌⠉⠬⠙⠼ $^{a + b}\diagup_{c + d}$

(a plus b, over c plus d; in print, a long diagonal, a + b is written to its left and near the top, c + d is written to its right and near the bottom)

(2) ⠼⠒⠹⠭⠸⠌⠽⠼ $3\ {\scriptstyle x/y}$

(3 times the fraction x over y; in print, the 3, x and y are at the same level, but the x and y are in smaller type than the 3)

c. To enclose a complex fraction whose numerator and denominator are separated by a horizontal line in print, provided that simple-fraction indicators are *not* used for any of the inner simple fractions.

(1) ⠹⠢⠌⠲⠸⠹⠒⠌⠦⠸⠼⠼ $\dfrac{5}{4\frac{3}{8}}$

(5 over the mixed number four and three eighths, all fraction lines horizontal in print)

(2) ⠹⠒⠸⠌⠲⠌⠢⠼ $\dfrac{3/4}{5}$

(three-fourths over 5, with diagonal line between 3 and 4, horizontal line between the three-fourths and the 5)

§31. Non-use of Simple-fraction Indicators: Simple-fraction indicators must not be used:

a. To enclose a simple fraction whose numerator and denominator are separated by a horizontal line, if the simple fraction is a fractional part of a mixed number.

(1) ⠼⠲⠸⠹⠒⠌⠦⠸⠼ $4\frac{3}{8}$

(4 and three-eighths, horizontal line in print)

(2) ⠼⠆⠸⠹⠒⠌⠲⠸⠼⠭ $2\frac{3}{4}x$

(2 and three-fourths x, horizontal line in print, x follows the mixed number)

b. To enclose a simple fraction whose numerator and denominator are separated by a diagonal line in print, provided that the expressions on either side of the diagonal line appear at the same level relative to it, *and* are of the same type size as the surrounding text.

(1) ⠼⠂⠸⠌⠒ $1/3$

(one-third, diagonal line in print)

(2) ⠭⠘⠂⠸⠌⠆ $x^{1/2}$

(x to the one-half power, diagonal line between the 1 and the 2)

(3) ⠭⠘⠹⠂⠌⠆⠼⠐⠸⠌⠆ $x^{\frac{1}{2}}/2$

(x to the one-half power, over 2; horizontal line in the superscript, diagonal line before the 2)

(4) ⠭⠘⠂⠸⠌⠆⠐⠸⠌⠆ $x^{1/2}/2$

(x to the one-half power, over 2; diagonal line in superscript, diagonal line before the 2)

(5) ⠁⠬⠃⠸⠌⠉⠬⠙ $a + b/c + d$

(a, plus b over c, plus d; diagonal line in print)

(6) ⠷⠁⠬⠃⠾⠸⠌⠷⠉⠬⠙⠾ $(a + b)/(c + d)$

(a plus b over c plus d, the numerator and denominator each in parentheses; diagonal line in print)

(7) ⠠⠊⠸⠌⠷⠝⠾ $I/(n)$

(I modulo n, diagonal line in print; a common expression in modern mathematics which is not a fraction at all)

§32. Mixed Number:

a. A mixed number is an expression which begins with a numeral and is followed, usually in smaller type, by a simple fraction whose numerator and denominator are both numerals. The fraction line of this simple fraction may be either horizontal or diagonal in print.

The mixed-number indicators must be used to enclose the fractional part of a mixed number.

(1) ⠼⠲⠸⠹⠒⠌⠦⠸⠼ $4\frac{3}{8}$

(4 and three-eighths, horizontal line in print)

(2) ⠼⠲⠸⠹⠒⠸⠌⠦⠸⠼ $4{}^3\!/_8$

(4 and three-eighths, diagonal line in print)

b. An expression is not a mixed number if it contains any letters, even though such an expression is of the same form as a mixed number in every other respect.

(1) ⠭⠹⠒⠌⠦⠼ $x\frac{3}{8}$

(x followed by the fraction three over eight, horizontal line in fraction, fraction in smaller type than the x)

(2) ⠼⠒⠹⠂⠸⠌⠽⠼ $3\ 1/y$

(3 followed by the fraction 1 over y, diagonal line in fraction, fraction in smaller type than the 3)

§33. Complex Fraction:

a. Except as noted in **b** below, a *complex fraction* is one whose numerator, denominator, or both, contains at least one simple fraction.

b. An expression is *not a complex fraction* if the only simple fractions it contains are at the subscript or superscript level.

§34. Use of Complex-fraction Indicators: Complex-fraction indicators must be used to enclose a complex fraction, provided that simple-fraction indicators are used for at least one of the inner simple fractions.

(1) ⠠⠹⠹⠒⠌⠦⠼⠠⠌⠢⠠⠼

$$\frac{\frac{3}{8}}{5}$$

(three eighths over 5, both fraction lines are horizontal in print)

§35. Non-use of Complex-fraction Indicators: Complex-fraction indicators must not be used to enclose a complex fraction if simple-fraction indicators are not used for any of the inner simple fractions; simple-fraction indicators must be used instead.

(1) ⠹⠂⠸⠌⠆⠌⠆⠸⠹⠆⠌⠒⠸⠼⠼

(one-half over two and two-thirds)

$$\frac{1/2}{2\frac{2}{3}}$$

(2) ⠹⠆⠸⠌⠒⠌⠒⠸⠌⠆⠼

(two-thirds over three-halves)

$$\frac{2/3}{3/2}$$

§36. Hypercomplex Fraction:

a. Except as noted below in **b**, a *hypercomplex fraction* is one whose numerator, denominator, or both, contain at least one complex fraction.

b. An expression is *not a hypercomplex fraction* if the only complex fractions it contains are at the subscript or superscript level.

(1) ⠹⠁⠌⠃⠘⠠⠹⠹⠒⠌⠲⠼⠠⠌⠹⠢⠌⠖⠼⠠⠼⠐⠼

$$\frac{a}{b^{\frac{\frac{3}{4}}{\frac{5}{6}}}}$$

(a over the expression b to the three-fourths over five-sixths power; the expression is not a hypercomplex fraction since its only complex fraction is at the superscript level)

§37. Use of Hypercomplex-fraction Indicators: Hypercomplex-fraction indicators must be used to enclose a hypercomplex fraction, and, in addition, a spatial arrangement must be used for such a fraction. The subsidiary fractions may be written either linearly or spatially (see Example 39a(1)).

(1)

```
           ⠙          ⠙
     ⠷⠂⠤⠭⠾⠹⠒⠒⠼⠷⠆⠭⠾⠤⠆⠭⠹⠒⠒⠼⠷⠂⠤⠭⠾
           ⠙⠭         ⠙⠭
   ⠠⠹⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠠⠼
              ⠷⠂⠤⠭⠾⠘⠆
⠠⠠⠹⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠠⠠⠼
                ⠼⠆⠭
           ⠼⠂⠬⠷⠹⠒⠒⠒⠒⠼⠾⠘⠆
                ⠼⠂⠤⠭
```

$$\frac{\dfrac{(1-x)\dfrac{d}{dx}(2x)-2x\dfrac{d}{dx}(1-x)}{(1-x)^2}}{1+\left(\dfrac{2x}{1-x}\right)^2}$$

§38. **Continued Fraction:** A *continued fraction* is one in which each denominator, except possibly the last one, is the sum of a whole number and a fraction. A spatial arrangement must be used for such a fraction.

(1)

```
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠼⠂
⠜⠆⠻⠀⠨⠅⠀⠼⠂⠬⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠼⠂
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠼⠆⠬⠒⠒⠒⠒⠒⠒⠒⠒⠒
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠼⠂
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠼⠆⠬⠒⠒⠒⠒⠒⠒
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠼⠂
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠼⠆⠬⠒⠒⠒
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠼⠆⠬⠀⠠⠠⠠⠸⠲
```

$$\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \ldots .}}}}$$

§39. **Use of Spatial Arrangement for Fractions** (see also §186): A spatial arrangement for a fraction must be used:

a. When the fraction is hypercomplex. In this situation, all fraction indicators must be shown, and each fraction line must have precisely the length necessary to cover the longest expression with which it is associated. (See §37(1) above for an example.)

The runover of an expression which is too long to be centered on the fraction line with which it is associated may be effected at suitable places in accordance with the rules for runovers (see §184), but each portion of the divided expression must be centered on the fraction line with which the expression, as a whole, is associated. The centering requirement is for the purpose of keeping the braille reader aware of standard print usage.

The use of linear arrangement within a spatial arrangement is permissible and, in fact, is to be preferred over the pure spatial arrangement, since considerable space and ease of transcription are gained without any loss of clarity for the braille reader.

(1)

```
   ⠠⠹⠷⠂⠤⠭⠾⠹⠙⠌⠙⠭⠼⠷⠆⠭⠾
    ⠤⠼⠆⠭⠹⠙⠌⠙⠭⠼⠷⠂⠤⠭⠾
     ⠠⠌⠷⠂⠤⠭⠾⠘⠆⠐⠠⠼
⠠⠠⠹⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠠⠠⠼
    ⠼⠂⠬⠷⠹⠆⠭⠌⠂⠤⠭⠼⠾⠘⠆
```

$$\frac{\dfrac{(1-x)\dfrac{d}{dx}(2x)-2x\dfrac{d}{dx}(1-x)}{(1-x)^{2}}}{1+\left(\dfrac{2x}{1-x}\right)^{2}}$$

(this is the mathematical equivalent of §37, example (1) above; it shows centering, runovers, and the use of linear arrangement within a spatial arrangement)

b. When the fraction is a continued fraction. In this situation, each fraction line must have proportionately the length shown in print. Fraction indicators of any kind must not be used with a continued fraction. (See §38(1) above for an example.)

c. When the fraction is one in which at least one word or abbreviation of measurement has been used in the role of a mathematical expression. In this situation, *all appropriate fraction indicators must be used* and all fraction lines must have the length necessary to span the longest expression with which it is associated.

(1)

```
         ⠲⠞⠨⠑
⠗⠁⠞⠑ ⠨⠅ ⠹⠒⠒⠒⠒⠼⠸⠲
          ⠐⠞
```

$$\text{rate} = \frac{\text{distance}}{\text{time}}.$$

(2)

```
 ⠼⠢⠆⠦⠴ ⠋⠞  ⠼⠖⠴ ⠍⠊
⠹⠒⠒⠒⠒⠒⠒⠒⠒⠼⠹⠒⠒⠒⠒⠒⠒⠼
  ⠼⠂ ⠍⠊    ⠼⠂ ⠓⠗
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
    ⠼⠂ ⠓⠗   ⠼⠂ ⠍⠊⠝
  ⠹⠒⠒⠒⠒⠒⠒⠒⠼⠹⠒⠒⠒⠒⠒⠒⠒⠼
   ⠼⠖⠴ ⠍⠊⠝  ⠼⠖⠴ ⠎⠑⠉
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
      ⠼⠦⠦ ⠋⠞
  ⠨⠅ ⠹⠒⠒⠒⠒⠒⠒⠼ ⠨⠅ ⠼⠦⠦ ⠋⠞⠸⠌⠎⠑⠉⠸⠲
      ⠼⠂ ⠎⠑⠉
```

$$\frac{5280\text{ ft}}{1\text{ mi}}\ \frac{60\text{ mi}}{1\text{ hr}}\ \frac{1\text{ hr}}{60\text{ min}}\ \frac{1\text{ min}}{60\text{ sec}}$$

$$=\frac{88\text{ ft}}{1\text{ sec}}=88\text{ ft/sec.}$$

§40. Permissive Use of Spatial Arrangement for Fractions: In general, the linear arrangement for fractions should be used when not expressly forbidden in §39. However, when fractions are first introduced, as in the lower grades, or when there is any other special need, any fraction may be represented spatially. The appropriate fraction indicator must be used in accordance with the rules in §§30-31, 32, 34-35, and 37, and the numeric indicators must be used in accordance with **§3b.**

(1)

```
 ⠼⠂⠬⠒
⠹⠒⠒⠒⠒⠼
 ⠼⠆⠬⠲
```

$$\frac{1+3}{2+4}$$

(2)

```
 ⠭
⠹⠒⠼
 ⠽
```

$$\frac{x}{y}$$

RULE V—OMISSIONS AND THE OMISSION INDICATOR

Omission Indicator ⠿ (dots 1-2-3-4-5-6)

§41. **Use of Omission Indicator:** The omission indicator must be used to represent any kind of an omission sign *only* when the reader is *explicitly* requested to replace the omission sign by an expression which is *mathematical.* This situation arises most frequently in the exercises of textbooks and workbooks. However, if the nature of the omission sign plays any role in supplying the correct answer, the corresponding braille symbol for that omission sign must be used instead.

(1) ⠼⠶⠤⠿ ⠨⠅ ⠼⠢ 7 — ? = 5

(it is understood that the reader has been specifically requested to replace the omitted numeral)

(2) ⠼⠶⠿⠆ ⠨⠅ ⠼⠢ 7 ? 2 = 5

(the reader is specifically requested to replace the omission sign by a sign of operation)

The number of omission indicators to be used is the same as the number of omission signs indicated in the printed text. When a space is used as the sign of omission, it must be replaced by one omission indicator. When an ellipsis or three small hyphens are used as omission signs in print, they must be regarded as a single sign and only one omission indicator used.

(3) ⠹⠿⠿⠌⠶⠆⠒⠼ $\frac{??}{723}$

(two question marks in print)

§42. **Non-use of Omission Indicator:** The omission indicator must not be used:

a. If the reader is *not* specifically requested to replace the omission sign by the omitted material.

(1) ⠼⠆⠠ ⠼⠲⠠ ⠼⠖⠠ ⠠⠠⠠⠸⠲ 2, 4, 6, . . .

b. When the omitted material is *literary.* In this situation, provided the reader has been specifically requested to replace the omitted material, the long dash of English Braille must be used instead of the omission indicator.

(1) ⠠⠮ ⠕⠏⠏⠕⠎⠊⠞⠑ ⠷ ⠁⠲⠊⠰⠝ ⠊⠎ ⠤⠤⠤⠤⠲

The opposite of addition is ——.

(2) ⠠⠋⠊⠧⠑ ⠈⠡ ⠹⠗⠑⠑ ⠨⠅ ⠤⠤⠤⠤⠲ Five × three = ——.

§43. **Punctuation of the Omission Indicator:** The omission indicator is a mathematical expression, and the punctuation and spacing used with it are the same as would be used with the mathematical expression which it replaces.

(1) ⠼⠒⠠ ⠼⠢⠠ ⠿⠠ ⠼⠔ 3, 5, . . . , 9

(ellipsis in print; the reader has specifically been requested to replace the missing numeral)

(2) ⠼⠆⠘⠿ ⠨⠅ ⠼⠖⠲ $2^{?} = 64$

(question mark is superscript in print)

(3) ⠼⠂⠆⠰⠤⠤⠤⠤ ⠨⠅ ⠼⠢ $12_{\Diamond} = 5$

(the reader has specifically been instructed to replace the omission by one of the words *three* or *seven*)

(4) ⠼⠢ ⠨⠅ ⠼⠂⠆⠰⠤⠤⠤⠤ $5 = 12_{?}$

(an English word is to be supplied)

RULE VI—CANCELLATION AND CANCELLATION INDICATORS

Cancellation Indicators

Opening ⠪ (dots 2-4-6)

Closing ⠻ (dots 1-2-4-5-6)

§44. Use of Cancellation Indicators:

a. The cancellation indicators must be used to show the extent of a mathematical expression which has been cancelled in print. A spatial arrangement must be used whenever cancellation is represented in braille. It is unnecessary to indicate the direction of the cancellation stroke. The use of a linear arrangement within a spatial arrangement is permissible, as is shown in examples 2 and 3 below. Cancellation for the purpose of reducing fractions should not be regarded as work arranged for computation.

(1)

```
 ⠼⠂
 ⠪⠭⠻
⠹⠒⠒⠒⠒⠼
 ⠪⠭⠻⠽
  ⠽
```

$$\frac{\overset{1}{\cancel{x}}}{\underset{y}{\cancel{x}}y}$$

(x cancelled in print)

(2)

⠀⠼⠂

⠀⠪⠢⠻

⠹⠒⠒⠒⠒⠼⠀⠀⠨⠅⠀⠀⠹⠂⠌⠢⠼

⠀⠪⠆⠢⠻

⠀⠀⠼⠢

$$\frac{\overset{1}{\cancel{5}}}{\underset{5}{\cancel{25}}} = \frac{1}{5}$$

(five twenty-fifths equal one-fifth)

(3)

⠀⠀⠀⠪⠷⠭⠬⠽⠾⠻

⠹⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠼⠀⠀⠨⠅⠀⠀⠹⠂⠌⠽⠬⠵⠼

⠀⠪⠷⠭⠬⠽⠾⠻⠷⠽⠬⠵⠾

$$\frac{\cancel{(x+y)}}{\cancel{(x+y)}\,(y+z)} = \frac{1}{y+z}$$

((x plus y) cancelled from numerator and denominator)

(4)

⠀⠀⠀⠦⠀⠀⠔⠀⠀⠔⠀⠀⠂⠆

⠀⠀⠪⠔⠻⠪⠴⠻⠪⠴⠻⠪⠀⠆⠻

⠀⠤⠀⠒⠀⠀⠖⠀⠀⠔⠀⠀⠀⠒

⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒

⠀⠀⠀⠢⠀⠀⠒⠀⠀⠴⠀⠀⠀⠔

$$\begin{array}{rrrrr} & 8 & 9 & 9 & 12\\ & \cancel{9} & \cancel{0} & \cancel{0} & \cancel{2}\\ - & 3 & 6 & 9 & 3\\ \hline & 5 & 3 & 0 & 9\end{array}$$

(9002 minus 3693 is 5309; this example shows how digits are cancelled in a subtraction example involving "borrowing" or "changing number form"; numeric indicators are not used because this example is arranged for computation (see §4d).)

b. The distinction between the cancellation stroke and the diagonal negation stroke is made by noting that the cancellation stroke is never applied to signs of comparison, whereas the negation stroke is never applied to mathematical expressions which contain letters or numerals.

RULE VII—CAPITALIZATION, ALPHABETS, TYPE FORMS, AND ASSOCIATED INDICATORS

Alphabetic Indicators

German letters	⠸	(dots 4-5-6)
Greek letters	⠨	(dots 4-6)
Hebrew letters	⠠⠠	(dots 6, 6)
Roman letters	⠰	(dots 5-6)
Russian (Cyrillic) letters	⠈⠈	(dots 4, 4)

Capitalization Indicator	⠠	(dot 6)

Type-form Indicators

Boldface	⠸	(dots 4-5-6)
Italic	⠨	(dots 4-6)
Sanserif	⠫⠎	(dots 1-2-4-6, 2-3-4)
Script	⠈	(dot 4)

Alphabets: See Appendix, page 123.

§45. Use of the Capitalization Indicator:

a. The capitalization indicator must be used to indicate the capitalization of a letter from any of the alphabets listed in the Appendix. This indicator must precede the alphabetic symbol concerned.

(1) ⠸⠠⠁ 𝔄

(German capitalized a)

(2) ⠨⠠⠁ Α

(Greek capitalized alpha)

b. The single capital symbol must be used before a Roman numeral consisting of a single capitalized letter, and the double capital symbol must be used before a Roman numeral consisting of more than one capitalized letter, provided that such Roman numerals are not used in formulas or for the purpose of performing computation, or when they do not carry primes, subscripts, superscripts, or modifiers. (See §10a, example (2).)

§46. Non-use of the Capitalization Indicator: Capitalization must not be used for a letter just because it begins a sentence, if the corresponding letter in print is uncapitalized.

(1) ⠰⠭ ⠊⠎ ⠁ ⠝⠥⠍⠃⠻ ⠆⠞ ⠼⠆ ⠯ ⠼⠒⠸⠲

x is a number between 2 and 3.

§47. Effectiveness of the Capitalization Indicator:

a. The effectiveness of the capitalization indicator extends only to the letter which follows it, so that if each letter in a sequence requires capitalization, the capitalization indicator must be used with each of these letters individually.

(1) ⠠⠁⠠⠃⠠⠉ ABC

(all letters are capitalized)

b. The double capital symbol is effective for all the letters of a Roman numeral which follow it. (See §10a, example (2).)

c. The capitalized words and letter sequences which are used in Fortran or other computer languages require special treatment, and no attempt is made to deal with such situations in the Rules of this Code.

(1) ⠠⠠⠗⠂⠙⠂ READ,

(the word *read* in full capitals, followed by a comma)

(2) ⠠⠠⠎⠊⠝⠋⠸⠲ SINF.

(the mathematical expression *sine function,* fully capitalized, unspaced and followed by a period)

§48. Alphabets:

a. Specific provision is made in this Code for five alphabets—German, Greek, Hebrew, Roman, and Russian (see Appendix). The letters of the Russian alphabet are often called *Cyrillic letters.* The alphabet to which a letter belongs is specified by the appropriate alphabetic indicator.

b. Letters from alphabets other than those for which specific provision has been made can also be represented. The entire representation is achieved by the shape indicator followed by two consecutive Roman lower-case letters which are suggestive of the name or the shape of the letter being represented. If the two consecutive letters constitute a contractable combination, the contraction must *not* be used. If such a letter is capitalized, the capitalization indicator must precede the shape indicator. A transcriber's note is also required to explain the letter combination which has been used. Such letters may also be drawn.

(1) ⠫⠎⠞ ϛ

(stigma, an obsolete Greek letter)

§49. Alphabetic Indicators: Except for the Roman-letter indicator, the rules concerning which are stated in §51, the appropriate alphabetic indicator must be used to specify the alphabet to which a letter belongs. If the letter is lower case, the corresponding alphabetic indicator precedes the letter directly; if the letter is capitalized, so that the capitalization indicator is also required, the alphabetic indicator must precede the capitalization indicator.

(1) ⠨⠁ α

(Greek lower-case alpha)

(2) ⠨⠠⠎ Σ

(Greek capitalized sigma)

(3) ⠨⠏ π

(Greek lower-case pi)

(4) ⠸⠁ $\mathfrak{a}$

(German lower-case a)

(5) ⠸⠠⠁ $\mathfrak{A}$

(German capitalized a)

(6) ⠠⠠⠁ $\aleph$

(Hebrew alef)

(7) ⠠⠠⠁⠴ $\aleph_0$

(alef sub zero)

(8) ⠈⠈⠁ а

(Russian lower-case ah)

(9) ⠈⠈⠠⠁ А

(Russian capitalized ah)

§50. **Effectiveness of Alphabetic Indicator:** The effectiveness of an alphabetic indicator extends only to the letter which follows it, so that (except for the Roman-letter indicator—see §51) an alphabetic indicator must be used with each individual letter of a sequence of letters.

(1) ⠨⠁⠨⠃ $\alpha\beta$

(Greek lower-case alpha followed by a Greek lower-case beta)

(2) ⠸⠠⠁⠨⠁⠬⠸⠠⠃⠨⠃ $\mathfrak{A}\alpha + \mathfrak{B}\beta$

(German capitalized a followed by Greek lower-case alpha, plus German capitalized b followed by Greek lower-case beta)

§51. Use of Roman-letter Indicator: The Roman-letter indicator must be used:

a. With Roman letters, whether lower-case or capitalized, of a type form other than ordinary type or sanserif.

(1) ⠸⠰⠠⠁⠸⠰⠠⠃ **AB**

(boldface Roman capitalized a followed by boldface Roman capitalized b)

(2) ⠨⠰⠁⠨⠰⠃ *ab*

(italic Roman lower-case a followed by italic Roman lower-case b)

(3) ⠈⠰⠑⠈⠰⠋ *ef*

(script Roman lower-case e followed by script Roman lower-case f)

b. With a single Roman letter or a sequence of Roman letters separated by commas, whether lower-case or capitalized, when such letters occur in a sentence in which the adjacent material is literary.

(1) ⠰⠭ ⠊⠎ ⠮ ⠔⠙⠑⠏⠢⠙⠢⠞ ⠧⠜⠊⠁⠼⠲

x is the independent variable.

(2) ⠰⠧ ⠙⠑⠝⠕⠞⠑⠎ ⠮ ⠝⠥⠍⠃⠻ ⠼⠑⠸⠲

v denotes the number 5.

(3) ⠰⠠⠁⠠ ⠰⠠⠃⠠ ⠰⠠⠉ ⠜⠑ ⠑⠟⠥⠊⠙⠊⠎⠞⠁⠝⠞ ⠋ ⠮ ⠉⠢⠞⠻⠲

A, B, C are equidistant from the center.

(4) ⠠⠇⠑⠞ ⠰⠙ ⠙⠑⠝⠕⠞⠑ ⠮ ⠲⠞⠨⠑⠲

Let d denote the distance.

(5) ⠠⠊⠋ ⠰⠠⠁ ⠯ ⠰⠠⠃ ⠜⠑ ⠮ ⠢⠙⠏⠕⠔⠞⠎ ⠷⠁ ⠇⠔⠑ ⠎⠑⠛⠰⠞⠂ ⠄⠄⠄

If A and B are the endpoints of a line segment, . . .

(6) ⠠⠮ ⠧⠻⠞⠊⠉⠑⠎ ⠷⠮ ⠎⠟⠥⠜⠑ ⠜⠑ ⠰⠠⠁⠠ ⠰⠠⠃⠠ ⠰⠠⠉⠠ ⠯ ⠰⠠⠙⠸⠲

The vertices of the square are A, B, C, and D.

(7) ⠠⠋⠔⠙ ⠮ ⠧⠁⠇⠥⠑ ⠷ ⠰⠭⠸⠲

Find the value of x.

c. With a single Roman letter, lower-case or capitalized, surrounded by quotation marks.

(1) ⠦⠰⠭⠸⠴ “x”
(lower-case x in outer quotation marks)

(2) ⠠⠦⠰⠠⠁⠸⠴⠄ ‘A’
(capitalized a in inner quotation marks)

d. With a single Roman letter, lower-case or capitalized, which is followed by an abbreviation of measurement.

(1) ⠠⠁ ⠗⠑⠉⠞⠁⠝⠛⠇⠑ ⠰⠓ ⠋⠞ ⠇⠰⠛
⠃⠽ ⠰⠅ ⠋⠞ ⠓⠊⠣⠲
A rectangle h ft long by k ft high.

e. Before lower-case Roman numerals which do not occur in formulas or in work arranged for computation, or which do not carry primes, subscripts, superscripts, or modifiers. (See §10a, example (1).)

§52. Non-use of the Roman-letter Indicator: The Roman-letter indicator must not be used:

a. For a Roman letter in ordinary type, whether lower-case or capitalized, if the letter is part of a longer mathematical expression. This applies to mathematical sequences of letters in literary sentences.

(1) ⠭ ⠨⠅ ⠼⠴ $x = 0$
(single letter is part of an equation)

(2) ⠉⠕⠎ ⠠⠁ $\cos A$
(cosine of A)

(3) ⠭⠘⠆ x^2
(x square; single letter carries a superscript)

(4) ⠭⠽ ⠨⠅ ⠼⠲ $xy = 4$
(each letter is part of an equation)

(5) ⠠⠞⠗⠊⠁⠝⠛⠇⠑⠎ ⠁⠃⠉ ⠯ ⠠⠁⠠⠃⠠⠉
⠜⠑ ⠎⠊⠍⠊⠇⠜⠲
Triangles abc and ABC are similar.

b. When Roman letters, lower-case or capitalized, are enclosed in mathematical symbols of grouping (see §148a).

(1) ⠷⠠⠁⠾ (A)
(capitalized letter A enclosed in mathematical parentheses)

(2) ⠨⠷⠭⠨⠾ $\{x\}$
(lower-case letter x enclosed in mathematical braces)

(3) ⠳⠽⠳ $|y|$
(lower-case letter y enclosed in vertical bars)

c. Before Roman numerals which occur in formulas or in work arranged for computation, or which carry primes, subscripts, superscripts, or modifiers.

§53. Type Form of Letters: Specific provision is made in this Code for five type forms—boldface, italic, ordinary, sanserif, and script. Except for ordinary type, the type form of a letter is specified by the appropriate type-form indicator.

§54. Use of Type-form Indicator: Except for ordinary type, the appropriate type-form indicator must be used to specify the type form of a letter, provided that the form of type employed in the printed text conveys mathematical significance (see §59a). Except for the sanserif type form, a type-form indicator must always be followed by an alphabetic indicator.

(1) ⠨⠰⠁ *a*
(italic Roman lower-case a)

(2) ⠨⠰⠠⠁ *A*
(italic Roman capitalized a)

(3) ⠨⠸⠁ *a*
(italic German lower-case a)

(4) ⠸⠰⠁ **a**
(boldface Roman lower-case a)

(5) ⠸⠰⠠⠁ **A**
(boldface Roman capitalized a)

(6) ⠸⠨⠁ **α**
(boldface Greek lower-case alpha)

(7) ⠸⠸⠁ 𝖆
(boldface German lower-case a)

(8) ⠸⠈⠈⠁ **а**
(boldface Russian lower-case ah)

(9) ⠈⠰⠁ 𝒶
(script Roman lower-case a)

(10) ⠈⠰⠠⠁ 𝒜
(script Roman capitalized a)

(11) ⠈⠸⠁ 𝔞
(script German lower-case a)

(12) ⠈⠠⠠⠁ א
(script Hebrew alef)

(13) ⠠⠫⠎⠓⠠⠓ 𝖧H
(capitalized sanserif H, followed by an ordinary capital H)

§55. Non-use of Type-form Indicator: A type-form indicator must not be used when a letter is printed in ordinary type.

§56. Effectiveness of Type-form Indicator: When a type-form indicator is used, its effectiveness extends (except in the case of numerals) only to the symbol which follows it. Thus, except for ordinary type, a type-form indicator must be used with each individual letter of a sequence of letters.

(1) ⠁⠨⠰⠃⠸⠰⠉⠈⠰⠙ a*b***c**𝒹

(an ordinary a, followed by an italic b, followed by a boldface c, followed by a script d—all Roman lower-case letters)

§57. Type Forms of Numerals:

a. Numerals which are printed in a type form other than ordinary type must be transcribed by using the type-form indicators described in §53, provided that the variant type form employed in the printed text conveys mathematical significance (see §59a). In this situation, the type-form indicator must precede the numeric indicator.

(1) ⠸⠼⠴ **0**

(zero in boldface type)

(2) ⠈⠼⠆ 𝒵

(script 2)

b. If a numeral to be transcribed by using type-form indicators contains more than one digit, the numeric indicator must be used only before the first digit at which a transition has been made from one type form to another. If the transition is to a type form other than ordinary type, the numeric indicator must be preceded by the appropriate type-form indicator. The effectiveness of the type-form indicator with numerals extends until there is a change in type form.

(1) ⠸⠼⠒⠲⠢ **345**

(345 in boldface type)

(2) ⠨⠼⠒⠨⠢ *3.5*

(3.5, in italic type)

(3) ⠸⠼⠒⠼⠲⠢ **3**45

(3 in boldface type, 4 and 5 in ordinary type)

(4) ⠨⠼⠒⠸⠼⠲⠼⠢ *3***4**5

(an italic 3, a boldface 4, and an ordinary 5)

§58. Boldface Type:

a. The boldface-type indicator must be used before operation, comparison, or shape symbols when the corresponding signs are in boldface type. In the case of shape symbols, the boldface-type indicator follows the shape indicator. (See also §59a.) For regular polygons (see §101**h**, example (3)), the numeric indicator is not used before the numeral, as called for in §3**d**.

(1) ⠸⠬ +

(plus sign in boldface type)

(2) ⠸⠈⠡ ×

(the multiplication cross in boldface type)

(3) ⠸⠨⠅ =

(equals in boldface type)

(4) ⠫⠸⠗ ▬

(a filled-in rectangle)

(5) ⠫⠸⠲ ■

(a filled-in square)

b. Boldface type, used in many texts to identify letters as vectors, must be preserved in the transcription. When vectors are identified by use of boldface type *as well as* by the use of arrows, the arrows themselves must be omitted from the transcription, but a transcriber's note must be included indicating their presence in the print copy.

§59. Non-use of Type-form Indicators:

a. Type forms other than ordinary type and *having no mathematical significance*, are often employed in texts to draw attention, particularly in the earlier grades. When this is the case, the practice *must not* be carried over to the transcription. This applies to entire sentences or passages as well as to words, single letters, or numerals.

b. However, when the author singles out a type form to convey a specific mathematical meaning, the type form must be preserved in the transcription.

c. When sentences or passages containing mathematical expressions are in a type form other than ordinary type, and where the type form has mathematical significance, the sentence or passage must be preceded by dots 6, 3, type-form indicator, and a space; and must be followed by a space, type-form indicator, dots 6, 3. Type-form indicators are not used within the sentence or passage unless some letters or numerals within the sentence or passage are in another type form. Sentences or passages which are entirely literary must be treated as in English Braille.

§60. Plural of Letters (see §15).

(1) ⠰⠠⠁⠸⠄⠎⠠ ⠰⠠⠃⠸⠄⠎⠠ ⠯ ⠰⠠⠉⠸⠄⠎⠸⠲

A's, B's, and C's.

(the plural of A, B, and C)

§61. Punctuation of Single Letters: Single letters of any type form, whether lower-case or capitalized, are mathematical expressions and must be punctuated accordingly.

(1) ⠠⠎⠕⠇⠧⠑ ⠿ ⠰⠭⠸⠲

Solve for x.

(2) ⠰⠽ ⠊⠎ ⠁ ⠋⠥⠝⠉⠰⠝ ⠷ ⠰⠥⠸⠆ ⠰⠥ ⠊⠎ ⠁
⠋⠥⠝⠉⠰⠝ ⠷ ⠰⠭⠸⠲

y is a function of u; u is a function of x.

(3) ⠠⠮ ⠰⠠⠭⠤⠠ ⠰⠠⠽⠤⠠ ⠯ ⠰⠠⠵⠤⠁⠭⠊⠎⠲

The X-, Y-, and Z-axis.

§62. **Letters with Ordinal Endings:** When the ordinal ending *th* is required after a letter, it must be adjoined directly to the letter. The *th* contraction must not be used. An ordinal ending is a mathematical expression and must be punctuated accordingly. (For numerals with ordinal endings, see §7.)

(1) ⠠⠮ ⠊⠞⠓ ⠗⠪ ⠯ ⠚⠞⠓ ⠉⠕⠇⠥⠍⠝⠲

The ith row and jth column.

(2) ⠠⠁⠝ ⠑⠟⠥⠠⠝ ⠷⠮ ⠝⠞⠓ ⠕⠗⠙⠻⠲

An equation of the nth order.

RULE VIII—SUBSCRIPTS, SUPERSCRIPTS, AND LEVEL INDICATORS

Level Indicators

Base Line	⠐	(dot 5)
Subscript	⠰	(dots 5-6)
Superscript	⠘	(dots 4-5)
Subscript to Subscript	⠰⠰	(dots 5-6, 5-6)
Subscript to Superscript	⠘⠰	(dots 4-5, 5-6)
Superscript to Subscript	⠰⠘	(dots 5-6, 4-5)
Superscript to Superscript	⠘⠘	(dots 4-5, 4-5)

Subscript to Subscript to Subscript	⠰⠰⠰	(dots 5-6, 5-6, 5-6)
Subscript to Subscript to Superscript	⠘⠰⠰	(dots 4-5, 5-6, 5-6)
Subscript to Superscript to Subscript	⠰⠘⠰	(dots 5-6, 4-5, 5-6)
Subscript to Superscript to Superscript	⠘⠘⠰	(dots 4-5, 4-5, 5-6)
Superscript to Subscript to Subscript	⠰⠰⠘	(dots 5-6, 5-6, 4-5)
Superscript to Subscript to Superscript	⠘⠰⠘	(dots 4-5, 5-6, 4-5)
Superscript to Superscript to Subscript	⠰⠘⠘	(dots 5-6, 4-5, 4-5)
Superscript to Superscript to Superscript	⠘⠘⠘	(dots 4-5, 4-5, 4-5)

§63. **Nature of Subscripts and Superscripts:** It is characteristic of mathematical expression to employ signs, usually in smaller type, which are elevated or depressed relative to a reference line called the *base line.* A sign which is elevated relative to the base line is called a *superscript;* one which is depressed relative to the base line is called a *subscript.*

§64. **Order of Subscripts and Superscripts:** Superscripts or subscripts may carry superscripts or subscripts of their own; the latter are then referred to as superscripts or subscripts *of second order* and are thus distinguished from the former, which are called superscripts or subscripts *of first order.* Second order superscripts or subscripts may, in turn, carry superscripts or subscripts of their own, which are then called superscripts or subscripts *of third order.* While it is theoretically possible for a superscript or subscript to be of order higher than the third, this situation rarely arises in practice.

§65. **Level Indicators:** A level indicator other than the base-line indicator identifies the symbols which follow it as representing a superscript or subscript. The *base-line* indicator identifies the symbols which follow it as representing signs on the base line. The degree of elevation or depression specified by a level indicator is *always with respect to the base line;* the symbol which precedes the indicator, if it represents a sign at some other level, plays no role in this regard.

§66. **Use of Level Indicators:** Except for the subscript indicator with numeric subscripts described in §70, a level indicator other than the base-line indicator must be used to show that the symbols which follow it represent a superscript or a subscript. (See §65.) The base-line indicator must be used to terminate the effect of a preceding level indicator whose effect has not been terminated in some other way.

§67. **Orientation by Level Indicator:**

a. The effect of a level indicator with one component is to direct the reader's attention upward or downward from the base line according as that component is the superscript or the subscript indicator.

(1) ⠭⠘⠆ x^2

(x squared)

(2) ⠽⠘⠒ y^3

(y cubed)

(3) ⠭⠘⠈⠼ x^*

(the asterisk is a superscript to x)

(4) ⠭⠘⠤⠆ x^{-2}

(x to the minus 2; minus 2 is a superscript to x)

(5) ⠭⠰⠁ x_a

(x sub a)

(6) ⠭⠰⠤⠆ x_{-2}

(x sub minus 2; minus 2 is a subscript to x)

b. The effect of a level indicator with two components may be analyzed as follows:

The first component directs the reader's attention upward or downward from the base line according as that component is, in itself, the superscript or the subscript indicator.

The second component then directs the reader's attention upward or downward from this new position according as the second component is, in itself, the superscript or the subscript indicator.

(1) ⠝⠘⠭⠘⠘⠽ n^{x^y}

(n to the x to the y; y is a superscript to x, the combination is a superscript to n)

(2) ⠭⠘⠝⠘⠰⠁ x^{n_a}

(x to the n sub a power; a is a subscript to n, the combination is a superscript to x)

(3) ⠭⠰⠝⠰⠘⠁ x_{n^a}

(a is a superscript to n, the combination is a subscript to x)

(4) ⠝⠰⠭⠰⠰⠽ n_{x_y}

(n sub x sub y; y is a subscript to x, the combination is a subscript to n)

c. The effect of a level indicator with three components may be analyzed as follows:

The first two components direct the reader's attention from the base line to the position described in **b** above.

The third component then directs the reader's attention upward or downward from this new position, according as that component is, in itself, the superscript or the subscript indicator.

(1) ⠝⠘⠭⠘⠘⠽⠘⠘⠘⠵ $n^{x^{y^{z}}}$

(n to the x to the y to the z; z is a superscript to y, the combination is a superscript to x, and the larger combination is a superscript to n)

(2) ⠝⠘⠭⠘⠘⠽⠘⠘⠘⠵⠘⠘⠘⠘ ⠠⠠⠠ $n^{x^{y^{z^{\cdot^{\cdot^{\cdot}}}}}}$

(the ellipsis indicates the presence of superscripts of increasingly higher order, the dots are printed obliquely)

(3) ⠭⠘⠽⠘⠘⠵⠘⠘⠰⠁ $x^{y^{z_{a}}}$

(x to the y to the z sub a; a is a subscript to z, the combination is a superscript to y, and the larger combination is a superscript to x)

(4) ⠭⠘⠽⠘⠰⠁⠘⠰⠘⠝ $x^{y_{a^{n}}}$

(n is a superscript to a, the combination is a subscript to y, and the larger combination is a superscript to x)

(5) ⠝⠘⠭⠘⠰⠁⠘⠰⠰⠚ $n^{x_{a_{j}}}$

(j is a subscript to a, the combination is a subscript to x, and the larger combination is a superscript to n)

(6) ⠭⠰⠁⠰⠘⠗⠰⠘⠘⠝ $x_{a^{r^{n}}}$

(n is a superscript to r, the combination is a superscript to a, and the larger combination is a subscript to x)

(7) ⠭⠰⠁⠰⠘⠝⠰⠘⠰⠃ $x_{a^{n_{b}}}$

(b is a subscript to n, the combination is a superscript to a, and the larger combination is a subscript to x)

(8) ⠭⠰⠏⠰⠰⠁⠰⠰⠘⠍ $x_{p_{a^{m}}}$

(m is a superscript to a, the combination is a subscript to p, and the larger combination is a subscript to x)

(9) ⠝⠰⠭⠰⠰⠽⠰⠰⠰⠵ $n_{x_{y_{z}}}$

(n sub x sub y sub z; z is subscript to y, the combination is a subscript to x, and the larger combination is a subscript to n)

(10) ⠝⠰⠭⠰⠰⠽⠰⠰⠰⠵⠰⠰⠰⠰ ⠠⠠⠠ $n_{x_{y_{z_{\cdot_{\cdot_{\cdot}}}}}}$

(the ellipsis indicates the presence of subscripts of increasingly higher order, the dots are printed obliquely)

d. The effect of a level indicator with more than three components may be analyzed in the manner suggested for level indicators with two or three components.

e. In an expression such as below, the equations to the right of the vertical bar must not be regarded as subscripts, and the whole expression must be arranged spatially.

(1) ⠋⠷⠭⠠ ⠽⠠ ⠵⠾⠠⠳

⠠⠳⠭ ⠨⠅ ⠁
⠠⠳⠽ ⠨⠅ ⠃
⠠⠳⠵ ⠨⠅ ⠉

⠨⠅ ⠋⠷⠁⠠ ⠃⠠ ⠉⠾⠸⠲

$$f(x, y, z)\Big|_{\substack{x = a\\ y = b\\ z = c}} = f(a, b, c).$$

§68. **Left Subscripts and Superscripts:** A superscript or subscript may occupy a position to the left, as well as to the right, of the sign with which it is associated. The words *left* or *right* are then used with the words *superscript* or *subscript* to make the distinction in position.

A right or left superscript or subscript is represented as such merely by preserving the relative horizontal positions of the superscript or subscript symbol and the symbol with which it is associated. Each must be preceded by its appropriate level indicator.

Left superscripts or subscripts of the third or higher order, although rare, are treated in the manner suggested by the examples below.

(1) ⠘⠭⠐⠝ ${}^{x}n$

(x is a left superscript to n)

(2) ⠘⠤⠐⠭ ${}^{-}x$

(the minus sign is a left superscript to x)

(3) ⠰⠭⠐⠝ ${}_{x}n$

(x is a left subscript to n)

(4) ⠰⠭⠐⠝⠰⠽ ${}_{x}n_{y}$

(x is a left subscript to n, y is a right subscript to n)

(5) ⠼⠂⠴⠘⠘⠤⠘⠲ $10^{{}^{-}4}$

(the minus sign is a left superscript to the 4, the combination is a right superscript to the 10)

(6) ⠘⠝⠘⠰⠁⠐⠭ ${}^{n_{a}}x$

(a is a right subscript to n, the combination is a left superscript to x)

(7) ⠘⠰⠁⠘⠝⠐⠭ ${}^{{}_{a}n}x$

(a is a left subscript to n, the combination is a left superscript to x)

(8) ⠰⠝⠰⠘⠁⠐⠭ ${}_{n^{a}}x$

(a is a right superscript to n, the combination is a left subscript to x)

(9) ⠰⠘⠁⠰⠝⠐⠭ ${}_{{}^{a}n}x$

(a is a left superscript to n, the combination is a left subscript to x)

(10) ⠰⠭⠰⠰⠽⠐⠝ ${}_{x_y}n$

(y is a right subscript to x, the combination is a left subscript to n)

(11) ⠰⠰⠽⠰⠭⠐⠝ ${}_{{}_y x}n$

(y is a left subscript to x, the combination is a left subscript to n)

§69. **Direct Subscripts and Superscripts:** A superscript or subscript which occupies, respectively, a position directly over or under the sign with which it is associated is called a *modifier* (see §77-80).

§70. **Numeric Subscripts:** The subscript indicator must not be used to indicate a numeric subscript provided that *all* of the following conditions hold:

a. The corresponding numeric sign must be a *right,* and not a left, subscript.

b. The corresponding numeric sign must be a subscript of *first order,* and not of higher order.

c. The sign with which the numeric subscript is associated must be a *letter* from any alphabet and in any type form, but must not be a numeral or any other sign. The letter may carry one or more primes or superscripts. (See also §74a(3) and §75(4).)

d. The numeric subscript must carry *no* superscript, subscript, or primes of its own.

e. The sign which carries the numeric subscript must carry no other right subscript.

(1) ⠭⠂ x_1

(x sub 1)

(2) ⠭⠂⠂ x_{11}

(x sub 1, 1; a double subscript, or x sub eleven)

(3) ⠸⠠⠁⠂ $\mathfrak{A}_1$

(German capitalized a sub 1)

(4) ⠭⠄⠂ x'_1

(x prime sub 1)

(5) ⠭⠄⠄⠆ x''_2

(x double prime sub 2)

(6) ⠰⠒⠐⠭ ${}_3x$

(3 is a left subscript to x; subscript indicator is required because condition a does not hold)

(7) ⠭⠰⠊⠰⠰⠂ x_{i_1}

(x sub i sub 1; 1 is a subscript of second order; the subscript indicator is required because condition **b** does not hold)

(8) ⠼⠂⠆⠰⠶ 12_7

(12 sub 7; subscript indicator is required because condition c does not hold)

(9) ⠷⠠⠉⠠⠕⠒⠾⠰⠆ $(CO_3)_2$

(the carbonate radical taken twice; the subscript indicator is required before the 2 because condition c does not hold)

(10) ⠭⠰⠂⠰⠰⠚ x_{1_j}

(x sub 1 sub j; subscript indicator is required because condition **d** does not hold)

(11) ⠭⠰⠆⠰⠘⠝ x_{2^n}

(n is a superscript to 2, and the combination is a subscript to x; the subscript indicator is required because condition **d** does not hold)

(12) ⠭⠰⠆⠄ $x_{2'}$

(x sub 2 prime; the subscript indicator is required because condition **d** does not hold)

(13) ⠭⠰⠆⠬⠅ x_{2+k}

(2 plus k is subscript to x; the subscript indicator is required because condition **e** does not hold)

(14) ⠭⠰⠂⠸⠌⠆ $x_{1/2}$

(x sub one-half, diagonal line between 1 and 2; subscript indicator is required because condition **e** does not hold)

(15) ⠰⠒⠐⠭⠂ ${}_3x_1$

(3 is a left subscript to x, 1 is a right subscript to x)

§71. Use of Base-line Indicator: The base-line indicator must be used:

a. To terminate the effect of a preceding level indicator whose effect has not been terminated in some other way. (In addition to the following examples, see also §68(1-4), §68(6-11), §70(6), and §70(15) above.)

(1) ⠭⠘⠆⠐⠬⠂ $x^2 + 1$

(x squared, plus 1)

(2) ⠭⠰⠁⠐⠬⠂ $x_a + 1$

(x sub a, plus 1)

(3) ⠷⠭⠘⠆⠐⠬⠽⠘⠆⠐⠾ $(x^2 + y^2)$

(x squared plus y squared, expression enclosed within parentheses; base-line indicator before the plus sign and the closing parenthesis)

b. Before the termination indicator, a closing fraction indicator, or an order-of-radical indicator when a level indicator is in effect and the expresson being terminated is on the base line and is not, as a whole, at the level specified by that level indicator.

(1) ⠜⠭⠘⠆⠐⠬⠽⠘⠆⠐⠻ $\sqrt{x^2 + y^2}$

(the square root of x squared plus y squared)

(2) ⠹⠂⠌⠭⠘⠆⠐⠼ $\frac{1}{x^2}$

(the simple fraction 1 over x squared)

(3) ⠠⠹⠙⠷⠹⠭⠌⠽⠼⠾⠠⠌⠂⠬⠷⠹⠭⠌⠽⠼⠾⠘⠆⠐⠠⠼

$$\frac{d\left(\frac{x}{y}\right)}{1 + \left(\frac{x}{y}\right)^2}$$

(the complex fraction whose numerator is the differential of the simple fraction x over y, and whose denominator is 1 plus the square of the simple fraction x over y)

(See also §97(12), for an example involving an order-of-radical indicator and §82 involving the binomial coefficient).

§72. **Non-use of Base-line Indicator:** The base-line indicator must not be used to return to the base line from a numeric subscript, if the subscript indicator was not used.

(1) ⠭⠂⠬⠂ $x_1 + 1$

(x sub 1, plus 1)

(2) ⠷⠭⠂⠽⠂⠬⠭⠆⠽⠆⠾ $(x_1y_1 + x_2y_2)$

(x sub 1 y sub 1 plus x sub 2 y sub 2, the entire expression enclosed within parentheses)

§73. Effectiveness of Level Indicator:

The effectiveness of a level indicator is terminated:

a. By another level indicator.

(1) ⠭⠰⠁⠐⠬⠽⠘⠆ $x_a + y^2$

(x sub a, plus y squared)

(2) ⠹⠑⠘⠭⠘⠘⠆⠐⠌⠆⠼ $\frac{e^{x^2}}{2}$

(e to the x squared power, over 2)

b. By the punctuation indicator or the comma.

(1) ⠭⠘⠆⠸⠲ $x^2.$

(x squared, period after x)

(2) ⠭⠘⠆⠠ $x^2,$

(x squared, comma after x)

(3) ⠭⠰⠊⠰⠠ ⠰⠚ $x_{i,\ j}$

(the combination i, followed by a comma, followed by a space, followed by j is a subscript to x; without the subscript indicator before the comma, the comma would be represented as being on a level with x, whereas it is actually on a level with i)

(4) ⠭⠰⠂⠰⠠ ⠰⠆ $x_{1,\ 2}$

(the combination of 1, followed by a comma, followed by a space, followed by 2 is a subscript to x; comment as in **b**(3) above)

c. By either of the cancellation indicators.

(1) ⠪⠭⠘⠆⠻ $\cancel{x^2}$

(x squared, cancelled)

(2) ⠭⠘⠆⠪⠽⠘⠆⠻ $x^2\cancel{y^2}$

(x squared y squared, y squared is cancelled)

d. Except as in **e** and **f** below, by the space.

(1) ⠑⠘⠭ ⠨⠅ ⠽ $e^x = y$

(e to the x power equals y; space terminates the effect of the superscript indicator, so that the base line indicator is not required before the equals symbol)

(2) ⠼⠆⠘⠭ ⠐⠅ ⠼⠒⠘⠭ $2^x < 3^x$

(2 to the x power is less than 3 to the x power; comment as in (1) above applied to the *less than* symbol)

The effectiveness of a level indicator is not terminated:

e. By the space which follows an abbreviated scientific word or phrase (see §166) provided the indicator was introduced *before* the abbreviation. This space may follow the abbreviated scientific word or phrase directly, or the latter may carry a superscript or subscript before the advent of the space. The symbol which follows the space assumes the same level of elevation or depression as the abbreviated scientific word or phrase itself.

(1) ⠑⠘⠎⠊⠝ ⠭ $e^{\sin x}$

(e to the sine of x power)

(2) ⠑⠘⠉⠕⠎⠘⠘⠆ ⠭ $e^{\cos^2 x}$

(e to the cosine squared of x power)

(3) ⠟⠘⠇⠕⠛⠘⠰⠟ ⠁ ⠨⠅ ⠁ $q^{\log_q\ a} = a$

(the second q is a subscript to log; the combination is a superscript to the first q; the first a is at the same level as log; the first q, the equals sign, and the last a are on the base line)

f. By the space on either side of the ellipsis. (In addition to the following example, see also §67c(2) and §67c(10) above.)

(1) ⠭⠘⠂⠬⠂⠸⠌⠆⠬⠂⠸⠌⠒⠬ ⠠⠠⠠ ⠬⠂⠸⠌⠝ $x^{1 + 1/2 + 1/3 + \ldots + 1/n}$

(the ellipsis, as well as all the other signs which follow the x, are superscripts of the first order)

(2) ⠏⠂⠘⠨⠁⠘⠰⠂⠐⠏⠆⠘⠨⠁⠘⠰⠆ $p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_r^{\alpha_r}$

⠐⠠⠠⠠ ⠏⠰⠗⠘⠨⠁⠘⠰⠗

(p sub 1 to the alpha sub 1 power, times p sub 2 to the alpha sub 2 power, ellipsis, p sub r to the alpha sub r power; the ellipsis is on the base line)

g. By the transition to another braille line, unless the first symbol on that braille line is a comparison symbol.

(1) ⠭⠘⠆⠐⠬⠽⠘⠆⠐⠬⠵⠘⠆ $x^2 + y^2 + z^2 = r^2$

⠨⠅ ⠗⠘⠆

(x squared plus y squared plus z squared equals r squared; the equals symbol, being a comparison symbol and the first symbol on the new braille line, terminates the effect of the superscript indicator on the preceding braille line)

(2) ⠷⠂⠤⠎⠊⠝⠘⠆ ⠭⠾⠘⠆ $(1 - \sin^2 x)^2 \cos^2 x$

⠐⠉⠕⠎⠘⠆ ⠭

(the square of 1 minus sine squared x times the cosine squared of x; the symbol at the beginning of the new line is not a comparison symbol, so that the effectiveness of the superscript symbol on the preceding line is not automatically terminated by the transition to the new line)

§74. Simultaneous Subscripts and Superscripts:

a. When a sign simultaneously carries a subscript and a superscript, *the subscript must be indicated first,* even if the subscript is numeric and does not require the subscript indicator.

(1) ⠭⠰⠁⠘⠝ x_a^n

(x carries simultaneously a subscript of a and a superscript of n)

(2) ⠰⠁⠘⠝⠐⠭ ${}_a^n x$

(x carries simultaneously a left subscript of a and a left superscript of n)

(3) ⠭⠂⠘⠆ x_1^2

(x sub 1 squared)

b. When the same mathematical expression carries a subscript and superscript which are not simultaneous, the relative horizontal positions of the signs must be retained in the transcription.

(1) ⠁⠘⠝ ⠰⠍ $a^n{}_m$

§75. Primes in Addition to Subscripts or Superscripts: When a mathematical expression carries one or more primes in addition to subscripts or superscripts, the primes must be indicated first.

(1) ⠭⠄⠰⠁ x'_a

(x prime sub a)

(2) ⠭⠄⠘⠆ x'^2

(x prime squared)

(3) ⠭⠄⠰⠁⠘⠃ x'^{b}_{a}

(x prime simultaneously carries a subscript of a and a superscript of b)

(4) ⠭⠄⠄⠂⠘⠒ x''^{3}_{1}

(x double prime sub 1 cubed; x″ simultaneously carries a subscript of 1 and a superscript of 3)

§76. Plural of Mathematical Expressions which End with a Subscript or Superscript:

(1) ⠉⠰⠊⠸⠄⠎ c_i’s

(the plural of c sub i)

(2) ⠭⠘⠆⠸⠄⠎ x^2’s

(the plural of x squared)

(3) ⠭⠂⠸⠄⠎ x_1’s

(the plural of x sub 1)

RULE IX—MODIFIERS AND MODIFICATION INDICATORS

Modification Indicators

Directly Over	⠣	(dots 1-2-6)
Directly Under	⠩	(dots 1-4-6)

Modifiers

Arc

Opening downward	⌢	⠫⠁	(dots 1-2-4-6, 1)
Opening upward	⌣	⠫⠄	(dots 1-2-4-6, 3)

Arrow

Barbed at both ends	⟷	⠫⠪⠒⠒⠕	(dots 1-2-4-6, 2-4-6, 2-5, 2-5, 1-3-5)
Barbed at left	⟵	⠫⠪⠒⠒	(dots 1-2-4-6, 2-4-6, 2-5, 2-5)

Barbed at left and dotted at right	⟵•	⠫⠪⠒⠒⠡	(dots 1-2-4-6, 2-4-6, 2-5, 2-5, 1-6)
Barbed at right	⟶	⠫⠒⠒⠕	(dots 1-2-4-6, 2-5, 2-5, 1-3-5)
Dotted at both ends	•——•	⠫⠡⠒⠒⠡	(dots 1-2-4-6, 1-6, 2-5, 2-5, 1-6)
Dotted at left (no barb)	•——	⠫⠡⠒⠒	(dots 1-2-4-6, 1-6, 2-5, 2-5)
Dotted at left and barbed at right	•⟶	⠫⠡⠒⠒⠕	(dots 1-2-4-6, 1-6, 2-5, 2-5, 1-3-5)
Dotted at right (no barb)	——•	⠫⠒⠒⠡	(dots 1-2-4-6, 2-5, 2-5, 1-6)
Bar			
Horizontal (macron)	—	⠱	(dots 1-5-6)
Vertical	|	⠳	(dots 1-2-5-6)
Caret (circumflex)	^	⠸⠣	(dots 4-5-6, 1-2-6)
Inverted	˅	⠸⠩	(dots 4-5-6, 1-4-6)
Left-pointing	<	⠰⠣	(dots 5-6, 1-2-6)
Right-pointing	>	⠰⠩	(dots 5-6, 1-4-6)
Dot	•	⠡	(dots 1-6)
Equilateral triangle	△	⠫⠒	(dots 1-2-4-6, 2-5)
Hollow dot	∘	⠨⠡	(dots 4-6, 1-6)
Question mark	?	⠸⠦	(dots 4-5-6, 2-3-6)
Tilde			
Extended	∿∿	⠈⠠⠱	(dots 4, 6, 1-5-6)
Simple	∼	⠈⠱	(dots 4, 1-5-6)

§77. **Modifier:** A modifier is a superscript or subscript which occupies, respectively, a position directly over or directly under the sign with which it is associated.

§78. **The Five-step Rule for Transcribing Modified Expressions:**

a. The multipurpose indicator ⠐ (dot 5) must be used before the expression with which the modifier is associated.

b. The expression with which the modifier is associated must follow the multipurpose indicator.

c. The indicator for *directly over* ⠣ (dots 1-2-6) or *directly under* ⠩ (dots 1-4-6) must follow the expression with which the modifier is associated according as the modifier is a superscript or a subscript.

d. The modifier symbol must follow the modification indicator used in **c.**

e. The termination indicator ⠻ (dots 1-2-4-5-6) must follow the modifier symbol and signifies the completion of the modified expression. The termination indicator terminates only the modified expression but does not affect the level at which the modified expression occurs.

(1) ⠐⠭⠣⠱⠻ $\overline{x}$

(x with superscribed bar)

(2) ⠐⠭⠩⠱⠻ $\underline{x}$

(x with subscribed bar)

(3) ⠼⠒⠨⠢⠐⠲⠣⠱⠻ $3.5\overline{4}$

(3.54, bar superscribed to the 4)

(4) ⠐⠭⠬⠽⠣⠱⠻ $\overline{x + y}$

(bar superscribed to x plus y)

(5) ⠐⠇⠊⠍⠩⠭ ⠫⠒⠒⠕ ⠼⠴⠻ ⠋⠷⠭⠾ $\lim\limits_{x \to 0} f(x)$

(the limit, as x approaches 0, of f of x)

(6) ⠐⠷⠐⠁⠣⠱⠻⠸⠰⠠⠁
⠬⠐⠃⠣⠱⠻⠸⠰⠠⠃⠾⠣⠱⠻ $\overline{(\overline{a}\mathbf{A} + \overline{b}\mathbf{B})}$

(a bar times boldface Roman capitalized a, plus b bar times boldface Roman capitalized b, the expression enclosed within parentheses, and a bar superscribed to the entire expression, including the parentheses)

§79. Modifiers of Higher Order:

a. A modifier of second order must be preceded by the double, and a modifier of the third order must be preceded by the triple, of the appropriate modification indicator. The termination indicator, however, must be used only once, after the last modifier symbol.

(1) ⠐⠭⠬⠽⠣⠱⠣⠣⠁ ⠨⠅ ⠼⠒⠻ $\overset{a = 3}{\overline{x + y}}$

(a equals 3 is superscribed to the bar, the combination is superscribed to x plus y)

(2) ⠐⠭⠬⠽⠩⠱⠩⠩⠁ ⠨⠅ ⠼⠒⠩⠩⠩⠃ ⠨⠅ ⠼⠆⠻ $\underset{\underset{b = 2}{a = 3}}{\underline{x + y}}$

(b equals 2 is subscribed to a = 3, the combination is subscribed to the bar, and the larger combination is subscribed to x plus y)

b. A modifier, to be of order higher than the first, must be associated with the same expression as a modifier of lower order. In §78e(6) above, the long bar is not a modifier of second order because no modifier of first order is associated with the same expression as the long bar.

c. A modifier of order higher than the third must be treated in the manner suggested in a (1) and (2) above.

§80. **Simultaneous Modifiers:** When a mathematical expression is simultaneously modified above and below, the modification from below must be indicated first. The termination indicator, however, must be used only once, after the last modifier symbol. If the modifiers involved are of order higher than the first, they are treated as described above.

(1) ⠐⠭⠬⠽⠩⠱⠣⠱⠻ $\overline{\underline{x + y}}$

(x plus y, with subscribed and superscribed bars)

(2) ⠐⠨⠠⠎⠩⠝ ⠨⠅ ⠼⠂⠣⠠⠿⠻⠹⠂⠌⠆⠘⠝⠐⠼ ⠨⠅ ⠼⠂

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$$

(the summation from n equals 1 to infinity of 1 over 2 to the n power equals 1; n equals 1 is subscribed to the Greek capitalized sigma, the infinity sign is superscribed to it)

(3) ⠐⠭⠬⠽⠩⠱⠩⠩⠁ ⠨⠅ ⠼⠒⠣⠱⠣⠣⠃ ⠨⠅ ⠼⠆⠻ $\overset{b = 2}{\overline{\underset{a = 3}{\underline{x + y}}}}$

(a equals 3 is subscribed to the bar, the combination is subscribed to x plus y; b equals 2 is superscribed to the bar, the combination is superscribed to x plus y)

§81. **Horizontal Bars, Equals Sign:** Parallel horizontal bars must not be regarded as the equals sign when they occur above or below a mathematical expression. Furthermore, the bar which is more remote from the mathematical expression which is modified must not be regarded as a modifier of second order; the double bar must be regarded as a single modifier.

(1) ⠐⠭⠣⠱⠱⠻ $\overset{=}{x}$

(double horizontal bar is superscribed to x)

(2) ⠐⠭⠩⠱⠱⠣⠱⠱⠻ $\underset{=}{\overset{=}{x}}$

(double horizontal bars subscribed and superscribed to x)

(3) ⠐⠭⠩⠱⠱⠣⠱⠻ $\underset{=}{\overline{x}}$

(double bar subscribed to x, single bar superscribed to x)

§82. **Binomial Coefficient:** The two expressions which constitute a binomial coefficient must be separated by the *directly under* indicator. The expression which follows the opening parenthesis and precedes the *directly-under* indicator corresponds to the upper sign in the binomial coefficient; the expression which follows the *directly-under* indicator and precedes the closing parenthesis corresponds to the lower sign of the binomial coefficient. When the upper sign ends at a level other than the base line, the base-line indicator (dot 5), must precede the *directly-under* indicator.

(1) ⠷⠝⠩⠅⠾ $\binom{n}{k}$

(the binomial coefficient with n as the upper sign and k as the lower sign)

(2) ⠷⠛⠰⠚⠐⠩⠁⠰⠚⠐⠾ $\binom{g_j}{a_j}$

(the binomial coefficient with g sub j as the upper sign and a sub j as the lower sign)

§83. **Modified Expressions in Subscripts and Superscripts:** If a modified expression is part or all of a right or left superscript or subscript, the multipurpose indicator must be preceded by the appropriate level indicator. This will automatically be the case if the modified expression occurs at the beginning of the superscript or subscript; but the appropriate level indicator must be restated if the modified expression occurs at an interior position of the superscript or subscript.

(1) ⠠⠁⠰⠐⠭⠣⠱⠻ $A_{\bar{x}}$

(x bar is a right subscript to A)

(2) ⠠⠁⠰⠐⠭⠣⠱⠻⠬⠰⠐⠽⠣⠱⠻ $A_{\bar{x}+\bar{y}}$

(x bar plus y bar is a right subscript to A; the subscript indicator is restated before the multipurpose indicator in the second term of the subscript)

§84. **Plural of Modified Expressions** (see §15).

(1) ⠐⠭⠣⠱⠻⠸⠄⠎ $\bar{x}$'s

(the plural of x bar)

§85. **Interior Modifiers with Signs of Shape:** If two or more interior modifiers, arranged vertically, occur inside the same basic sign of shape, none of the interior signs may be regarded as a modifier of any of the others, and the technique for representing modified expressions described above does not apply. (See §103c.)

(1) ⠫⠉⠨⠫⠒⠒⠕⠫⠪⠒⠒

(a circle with an interior right-pointing arrow above a left-pointing arrow)

§86. **Arc:**

a. **Arc opening downward:**

(1) ⠐⠠⠁⠩⠫⠁⠻ $\underset{\frown}{A}$

(arc, opening downward, is subscribed to Roman capitalized a)

(2) ⠐⠠⠁⠠⠃⠣⠫⠁⠻ $\overset{\frown}{AB}$

(arc, opening downward, over AB)

b. **Arc opening upward:**

(1) ⠐⠠⠁⠩⠫⠄⠻ $\underset{\smile}{A}$

(arc opening upward under A)

(2) ⠐⠠⠁⠠⠃⠩⠫⠄⠻ $\underset{\smile}{AB}$

(arc opening upward under AB)

§87. **Arrows:** The arrows in the list at the beginning of this rule, as well as the horizontal bar, are frequently found in modern high school geometry courses. They are used to make careful distinction as to whether the end-points of a line are included or excluded from the line. These arrows, as well as the horizontal bar, must not be regarded as modifiers when they occur directly over or directly under a comparison sign. In that event, they become a component of a sign of comparison compounded vertically. (See §133.)

a. **Arrow barbed at both ends:** In geometry this arrow indicates a *full line.*

(1) ⠐⠠⠁⠠⠃⠣⠫⠪⠒⠒⠕⠻ $\overleftrightarrow{AB}$

(arrow barbed at both ends over AB)

b. **Arrow barbed at left:** In geometry, this arrow indicates a *half line* extending to the left.

(1) ⠐⠠⠁⠠⠃⠣⠫⠪⠒⠒⠻ $\overleftarrow{AB}$

(arrow barbed at left over AB)

c. **Arrow barbed at left and dotted at right:** In geometry this arrow indicates a *ray* extending to the left.

(1) ⠐⠠⠁⠠⠃⠣⠫⠪⠒⠒⠡⠻ $\overset{\longleftarrow\bullet}{AB}$

(arrow barbed at the left and dotted at the right over AB)

d. **Arrow barbed at right:** In geometry, this arrow indicates a *half line* extending to the right.

(1) ⠐⠠⠁⠠⠃⠣⠫⠒⠒⠕⠻ $\overrightarrow{AB}$

(arrow barbed at the right over AB)

e. **Arrow shaft dotted at both ends:** In geometry this arrow indicates a *closed interval* or *line segment.*

(1) ⠐⠠⠁⠠⠃⠣⠫⠡⠒⠒⠡⠻ $\overset{\bullet\!-\!\!-\!\!-\!\bullet}{AB}$

(arrow dotted at both ends over AB)

(2) ⠐⠠⠁⠠⠃⠣⠫⠨⠡⠒⠒⠨⠡⠻ $\overset{\circ\!-\!-\!\circ}{AB}$

(arrow with hollow dot at both ends over AB, open interval)

f. **Arrow shaft dotted at left (no barb at right):** In geometry, this arrow indicates a *half-open interval* closed at the left and open at the right.

(1) ⠐⠠⠁⠠⠃⠣⠫⠡⠒⠒⠻ $\overset{\bullet\!-\!-}{AB}$

(arrow dotted at left and no barb at right over AB)

g. **Arrow dotted at left and barbed at right:** In geometry, this arrow indicates a *ray* extending to the right.

(1) ⠐⠠⠁⠠⠃⠣⠫⠡⠒⠒⠕⠻ $\overset{\bullet\!\longrightarrow}{AB}$

(arrow dotted at the left and barbed at the right over AB)

h. **Arrow shaft dotted at right (no barb at the left):** In geometry, this arrow indicates a *half-open interval* closed at the right and open at the left.

(1) ⠐⠠⠁⠠⠃⠣⠫⠒⠒⠡⠻ $\overset{-\!-\!\bullet}{AB}$

(arrow shaft dotted at right and no barb at left over AB)

§88. **Horizontal Bar** (See §78):

a. In geometry, the horizontal bar indicates an *open interval.*

(1) ⠐⠠⠁⠠⠃⠣⠱⠻ $\overline{AB}$

(horizontal bar over AB)

b. When the horizontal bar is itself modified by a dot or a caret directly over or under it, the combination is a modified sign of comparison (see §132).

c. The horizontal bar is often used to indicate the recurrence of one or more digits in a decimal numeral by placing it over the digits which recur.

(1) ⠼⠨⠐⠒⠣⠱⠻ $.\overline{3}$

(decimal point 3, with a bar over the 3)

(2) ⠼⠨⠐⠶⠂⠆⠦⠣⠱⠻ $.\overline{7128}$

(decimal point 7128, with a bar over these four digits)

(3) ⠼⠒⠨⠢⠶⠐⠆⠔⠣⠱⠻ $3.57\overline{29}$

(3.5729, with a bar over the 29)

(4) ⠼⠒⠨⠢⠶⠐⠆⠔⠣⠱⠻ ⠄⠄⠄ $3.57\overline{29}\ldots$

(3.5729, with a bar over the 29, followed by an ellipsis)

d. When the double or triple bar is a modifier of a letter or number, it must not be mistaken for the equals sign or the identity sign (see §81).

(1) ⠐⠭⠣⠱⠱⠻ $\overset{=}{x}$

(x with a superscribed double bar)

(2) ⠐⠭⠩⠱⠱⠱⠻ $\underset{\equiv}{x}$

(x with a subscribed triple bar)

e. When the horizontal bar occurs over or under the integral sign, or over or under the abbreviated scientific word *lim,* or the fully expressed word *limit,* the bar must not be treated as a modifier (see §159 and §169).

§89. Vertical bar:

(1) ⠐⠨⠅⠣⠳⠻ $\overset{|}{=}$

(equals sign with vertical bar over it)

§90. Caret:

a. Ordinary caret:

(1) ⠐⠭⠣⠸⠣⠻ $\hat{x}$

(caret over x)

(2) ⠐⠨⠅⠣⠸⠣⠻ $\overset{\wedge}{=}$

(caret over equals sign)

b. Inverted caret:

(1) ⠐⠭⠩⠸⠩⠻ $\underset{\vee}{x}$

(inverted caret under x)

c. Left-pointing caret:

(1) ⠐⠨⠅⠣⠰⠣⠻ $\overset{<}{=}$

(left-pointing caret over the equals sign)

d. Right-pointing caret:

(1) ⠐⠨⠅⠣⠰⠩⠻ $\overset{>}{=}$

(right-pointing caret over the equals sign)

§91. Dot:

a. The dot is frequently used to indicate the recurrence of one or more digits in a decimal numeral. When used for this purpose, a dot is placed in print over each digit of the recurring sequence.

(1) ⠼⠨⠐⠒⠣⠡⠻ $.\dot{3}$

(decimal point 3, with dot over the 3)

(2) ⠼⠨⠐⠂⠒⠢⠣⠡⠻ $.\dot{1}\dot{3}\dot{5}$

(decimal point 135, with a dot over each of the three digits)

(3) ⠼⠨⠂⠒⠐⠢⠣⠡⠻ $.13\dot{5}$

(decimal point 135, with a dot over the 5)

b. Although there is theoretically no limit to the number of dots which may be placed over a single mathematical expression, in practice the number rarely exceeds three dots. However, as many dots must be used in the transcription as are present in the printed text, except in case of recurring decimals as in a.

(1) ⠐⠭⠣⠡⠻ $\dot{x}$

(one dot over x)

(2) ⠐⠭⠩⠡⠻ $\underset{\cdot}{x}$

(one dot under x)

(3) ⠐⠭⠣⠡⠡⠻ $\ddot{x}$

(two dots over x)

(4) ⠐⠭⠣⠡⠡⠡⠻ $\dddot{x}$

(three dots over x)

§92. Equilateral triangle:

In chemistry, this sign frequently occurs under an arrow, and the combination means *in the presence of heat.* The triangle is intended to be suggestive of the flame of a Bunsen burner.

(1) ⠐⠫⠒⠒⠕⠩⠫⠒⠻ $\overset{\rightarrow}{\triangle}$

(equilateral triangle under arrow barbed at the right)

(2) ⠐⠫⠒⠒⠕⠩⠫⠸⠒⠻ $\overset{\rightarrow}{\blacktriangle}$

(filled-in equilateral triangle under arrow barbed at the right)

(3) ⠐⠨⠅⠣⠫⠒⠻ $\overset{\triangle}{=}$

(equilateral triangle over equals sign)

§93. Hollow dot: When the hollow dot is placed over the equals sign, the combination usually means *is equal in degrees to.*

(1) ⠐⠨⠅⠣⠨⠡⠻ $\overset{\circ}{=}$

(hollow dot over the equals sign)

§94. Question mark:

(1) ⠐⠨⠅⠣⠸⠦⠻ $\overset{?}{=}$

(question mark over equals sign)

(2) ⠐⠨⠅⠩⠸⠦⠻ $\underset{?}{=}$

(question mark under equals sign)

§95. Tilde:

The tilde, simple or extended, must not be regarded as a modifier when it occurs directly over or directly under a comparison sign. In that event, it becomes a component of a sign of comparison compounded vertically (see §133).

When the tilde, simple or extended, is itself modified by a dot or a caret directly over or under it, the combination is a modified sign of comparison (see §132).

(1) ⠐⠭⠬⠽⠣⠈⠱⠻ $\widetilde{x + y}$

(tilde over x plus y)

(2) ⠐⠭⠬⠽⠬⠵⠣⠈⠠⠱⠻ $\widetilde{x + y + z}$

(extended tilde over x plus y plus z)

RULE X—THE MULTIPURPOSE INDICATOR

Multipurpose Indicator ⠐ (dot 5)

§96. **Use of the Multipurpose Indicator:** The multipurpose indicator must be used in the situations below for the specific purposes described and, when used in these situations, it must not be regarded as the base-line indicator:

a. Before a modified expression, as an indication to the reader of impending modification. (See §77-84 for additional information and examples.)

b. Between an alphabetic symbol and a succeeding numeric symbol, to indicate that the corresponding numeral is not a subscript to the corresponding letter.

(1) ⠭⠐⠢ x5

(x followed by 5)

c. Between two symbols of comparison, to indicate that the corresponding signs of comparison are printed *horizontally,* and not vertically. (See §134 and list in rule XVI.)

d. After the decimal point symbol, to indicate that the symbol which follows it is *not* numeric.

(1) ⠼⠴⠨⠐⠁⠂⠁⠆ ⠠⠠⠠ $0.a_1\ a_2 \ldots$

(0, decimal point, a sub 1, a sub 2, ellipsis)

(2) ⠼⠴⠨⠐⠨⠁⠂⠨⠁⠆ ⠠⠠⠠ $0.\alpha_1\ \alpha_2 \ldots$

(0, decimal point, alpha sub 1, alpha sub 2, ellipsis)

e. Between successive symbols of shape modification, to indicate that the corresponding shape modifiers are printed *horizontally.* (See subsection 103b for additional information.)

(1) ⠫⠉⠨⠳⠐⠱⠐⠳

(a circle with an interior vertical bar, followed by a horizontal bar, followed by another vertical bar; the interior modifiers are arranged horizontally and are separated from one another)

RULE XI—RADICALS AND RADICAL INDICATORS

Radical (square root) ⠜ (dots 3-4-5) $\sqrt{\qquad}$

Radical Indicators

Index-of-Radical Indicator ⠣ (dots 1-2-6)

Order-of-Radical Indicators

First Inner Radical ⠨ (dots 4-6)

Second Inner Radical ⠨⠨ (dots 4-6, 4-6)

Third Inner Radical ⠨⠨⠨ (dots 4-6, 4-6, 4-6)

Termination Indicator ⠻ (dots 1-2-4-5-6)

§97. Six-step Rule for Transcribing Radicals:

a. One of the order-of-radical indicators is used first, provided that the radical under consideration is one of the *inner* radicals of a nest. The order of the inner radical dictates the selection of the corresponding order-of-radical indicator. Provision is made specifically for order-of-radical indicators as high as the third order. Inner radicals of higher order are rare, but the corresponding order-of-radical indicator may be formed in the manner suggested in the above list of such indicators. The outer radical of a nest, or a simple radical belonging to no nest, does not require an order-of-radical indicator.

b. The index-of-radical indicator, followed by the index of the radical, is next, provided that an index is actually present in the printed text. If no index is present in the printed text, the radical is understood to represent the square root; in this case, no index-of-radical indicator and, of course, no index of radical, are required.

c. The radical symbol then follows. If the radical under consideration is unindexed, and if it is either a simple radical belonging to no nest or an outer radical of a nest, the radical symbol will be the first braille symbol to appear in the transcription of the radical.

d. The expression affected by the radical sign, called the radicand, is then transcribed directly following the radical symbol.

e. One of the order-of-radical indicators is again used, provided that such an indicator was also used to introduce the radical. The order-of-radical indicators associated in this way must indicate the same order. If no order-of-radical indicator was used to introduce the radical, none must be used following the radicand.

f. The termination indicator then follows, and signifies the completion of the radical under consideration.

(1) ⠜⠆⠻ $\sqrt{2}$

(the square root of 2)

(2) ⠜⠭⠬⠽⠻ $\sqrt{x+y}$

(the square root of x plus y)

(3) ⠜⠭⠘⠆⠐⠬⠂⠻ $\sqrt{x^2+1}$

(the square root of x squared plus 1)

(4) ⠜⠭⠘⠆⠐⠬⠽⠘⠆⠐⠻ $\sqrt{x^2+y^2}$

(the square root of x squared plus y squared)

(5) ⠜⠹⠭⠌⠽⠼⠻ $\sqrt{\frac{x}{y}}$

(the square root of the simple fraction x over y)

(6) ⠼⠒⠜⠁⠻ $3\sqrt{a}$

(3 times the square root of a)

(7) ⠣⠒⠜⠆⠻ $\sqrt[3]{2}$

(the cube root of 2)

(8) ⠼⠒⠣⠒⠜⠭⠬⠽⠻ $3\sqrt[3]{x+y}$

(3 times the cube root of x plus y)

(9) ⠣⠝⠜⠁⠻ $\sqrt[n]{a}$

(the nth root of a)

(10) ⠣⠍⠬⠝⠜⠏⠬⠟⠻ $\sqrt[m+n]{p+q}$

(the m plus nth root of p plus q)

(11) ⠜⠭⠬⠨⠜⠭⠬⠽⠨⠻⠬⠽⠻ $\sqrt{x + \sqrt{x + y} + y}$

(the square root of the sum of three terms; the first term is x, the second term is the square root of x plus y, and the third term is y)

(12) ⠣⠒⠜⠭⠘⠆⠐⠬⠨⠣⠒⠜⠭⠘⠆⠐⠬⠽⠘⠆⠐⠨⠻⠬⠽⠘⠆⠐⠻

$$\sqrt[3]{x^2 + \sqrt[3]{x^2 + y^2} + y^2}$$

(the cube root of the sum of x squared, the cube root of x squared plus y squared, and y squared)

(13) ⠜⠨⠣⠒⠜⠭⠨⠻⠻ ⠨⠅ ⠣⠒⠜⠨⠜⠭⠨⠻⠻ $\sqrt{\sqrt[3]{x}} = \sqrt[3]{\sqrt{x}}$

(the square root of the cube root of x equals the cube root of the square root of x)

(14) ⠜⠭⠬⠨⠜⠽⠬⠨⠨⠜⠵⠨⠨⠻⠨⠻⠻ $\sqrt{x + \sqrt{y + \sqrt{z}}}$

(a nest of three radicals; the outer radical contains x plus the inner radicals, the first inner radical contains y plus the second inner radical, and the second inner radical contains z)

(15) ⠜⠁⠴⠬⠨⠜⠁⠂⠬⠨⠨⠜⠁⠆⠬⠨⠨⠨⠜⠁⠒
⠬⠨⠨⠨⠨⠜ ⠠⠠⠠ ⠨⠨⠨⠨⠻⠨⠨⠨⠻⠨⠨⠻⠨⠻⠻

$$\sqrt{a_0 + \sqrt{a_1 + \sqrt{a_2 + \sqrt{a_3 + \sqrt{\cdots}}}}}$$

(an infinite nest of radicals; the outer radical contains a sub 0 plus the inner radicals, the first inner radical contains a sub 1 plus the remaining inner radicals, the second inner radical contains a sub 2 plus the remaining inner radicals, the third inner radical contains a sub 3 plus the remaining inner radicals, the fourth inner radical contains the ellipsis to indicate the continuation of the nest)

RULE XII—THE ENLARGEMENT INDICATOR AND SYSTEMS OF MATHEMATICAL EXPRESSIONS

Enlargement Indicator ⠠ (dot 6)

Enlarged Marks of Grouping

Braces

Left $\{$ ⠨⠠⠷ (dots 4-6, 6, 1-2-3-5-6)

Right $\}$ ⠨⠠⠾ (dots 4-6, 6, 2-3-4-5-6)

Brackets

Left [⠈⠠⠷ (dots 4, 6, 1-2-3-5-6)

Right] ⠈⠠⠾ (dots 4, 6, 2-3-4-5-6)

Parentheses

Left	(	⠠⠷	(dots 6, 1-2-3-5-6)
Right	)	⠠⠾	(dots 6, 2-3-4-5-6)

Vertical Bars

Double	$\|\|$	⠠⠳⠠⠳	(dots 6, 1-2-5-6, 6, 1-2-5-6)
Single	$\|$	⠠⠳	(dots 6, 1-2-5-6)

§98. **Use of Enlargement Indicator:** When a system of mathematical expressions is arranged on two or more lines of print, and a sign of grouping is used to unify the system, the corresponding grouping symbol in the transcription must be indicated as *enlarged.* Among such systems of mathematical expressions are: systems of equations, determinants, and matrices. Each braille line which contains any part of the transcription of such a system must also contain the representation of the enlarged sign of grouping. If only the left or only the right member of a pair of grouping signs is present in print, only the corresponding half of the grouping symbol must be represented in the transcription. However, when it seems advisable for any reason to do so, the enlarged grouping symbols may be drawn.

(1) ⠨⠠⠷⠭⠬⠽ ⠨⠅ ⠼⠆⠨⠠⠾⠸⠲
⠨⠠⠷⠭⠤⠽ ⠨⠅ ⠼⠴⠨⠠⠾

$$\left\{\begin{array}{l} x + y = 2 \\ x - y = 0 \end{array}\right\}.$$

(A two-line system of equations enclosed within braces)

(2) ⠠⠳⠁ ⠃⠠⠳ ⠨⠅ ⠁⠙⠤⠃⠉⠸⠲
⠠⠳⠉ ⠙⠠⠳

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

(The determinant with first-row entries of a and b, and second-row entries of c and d equals ad minus bc)

(3) ⠽ ⠨⠅ ⠈⠠⠷⠭⠠ ⠊⠋ ⠭ ⠐⠅⠨⠅ ⠼⠴
⠈⠠⠷⠴⠠ ⠊⠋ ⠭ ⠨⠂ ⠼⠴⠸⠲

$$y = \left[\begin{array}{l} x, \text{ if } x \leqq 0 \\ 0, \text{ if } x > 0. \end{array}\right.$$

(A two-line system unified on the left by a left bracket)

(4) ⠈⠠⠷⠉⠕⠎ ⠨⠁ ⠎⠊⠝ ⠨⠁ ⠴⠈⠠⠾
⠭ ⠨⠅ ⠈⠠⠷⠤⠎⠊⠝ ⠨⠁ ⠉⠕⠎ ⠨⠁ ⠴⠈⠠⠾
⠈⠠⠷⠴ ⠴ ⠂⠈⠠⠾

$$x = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(x is equal to a three-by-three matrix within large brackets)

(5) ⠠⠳⠠⠳⠂ ⠴ ⠴⠠⠳⠠⠳
⠠⠳⠠⠳⠴ ⠂ ⠴⠠⠳⠠⠳ ⠨⠅ ⠠⠊⠸⠲
⠠⠳⠠⠳⠴ ⠴ ⠂⠠⠳⠠⠳

$$\left\| \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \right\| = I.$$

(a three-by-three matrix enclosed within double vertical bars is equal to I; the matrix shown is the identity matrix with 1's along the diagonal and 0's elsewhere)

§99. **Non-use of Enlargement Indicator:** The enlargement indicator must not be used:

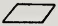
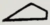
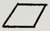
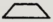
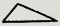
a. If a single line of print which is enclosed within signs of grouping requires more than one line of braille for its transcription.

b. If a fraction is enclosed within large signs of grouping.

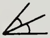
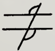
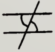
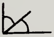
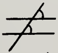
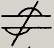
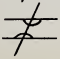
c. Before the radical or integral symbols when the corresponding signs in print are made large for the purpose of covering a fraction.

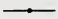
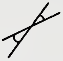
RULE XIII—SHAPES AND SHAPE INDICATORS

Shape Indicator		⠫	(dots 1-2-4-6)
Shape-modification Indicator		⠨	(dots 4-6)
Superposition Indicator		⠈	(dot 4)
Basic Shapes			
Angle	∠	⠫⠪	(dots 1-2-4-6, 2-4-6)
Arc			
Opening downward	⌒	⠫⠁	(dots 1-2-4-6, 1)
Opening upward	‿	⠫⠄	(dots 1-2-4-6, 3)
Circle	○	⠫⠉	(dots 1-2-4-6, 1-4)
Diamond	◇	⠫⠙	(dots 1-2-4-6, 1-4-5)
Ellipse (oval)	⬭	⠫⠑	(dots 1-2-4-6, 1-5)
Equilateral Triangle	△	⠫⠒	(dots 1-2-4-6, 2-5)
Hexagon			
Irregular	⬡	⠫⠓⠹	(dots 1-2-4-6, 1-2-5, 1-3-4-6)
Regular	⬡	⠫⠖	(dots 1-2-4-6, 2-3-5)

Intersecting lines	×	⠫⠊	(dots 1-2-4-6, 2-4)
Is parallel to	∥	⠫⠇	(dots 1-2-4-6, 1-2-3)
Is perpendicular to	⊥	⠫⠏	(dots 1-2-4-6, 1-2-3-4)
Parallelogram		⠫⠛	(dots 1-2-4-6, 1-2-4-5)
Pentagon			
Irregular		⠫⠏⠛	(dots 1-2-4-6, 1-2-3-4, 1-2-4-5)
Regular		⠫⠢	(dots 1-2-4-6, 2-6)
Quadrilateral		⠫⠟	(dots 1-2-4-6, 1-2-3-4-5)
Rectangle		⠫⠗	(dots 1-2-4-6, 1-2-3-5)
Rhombus	 or	⠫⠓	(dots 1-2-4-6, 1-2-5)
Square		⠫⠲	(dots 1-2-4-6, 2-5-6)
Trapezoid		⠫⠵	(dots 1-2-4-6, 1-3-5-6)
Triangle (scalene triangle)		⠫⠞	(dots 1-2-4-6, 2-3-4-5)

Shapes with Structural Modification

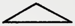
Angle			
Acute angle		⠫⠪⠨⠁	(dots 1-2-4-6, 2-4-6, 4-6, 1)
Adjacent angles		⠫⠪⠨⠚	(dots 1-2-4-6, 2-4-6, 4-6, 2-4-5)
Alternate exterior angles		⠫⠪⠨⠁⠑	(dots 1-2-4-6, 2-4-6, 4-6, 1, 1-5)
Alternate interior angles		⠫⠪⠨⠁⠊	(dots 1-2-4-6, 2-4-6, 4-6, 1, 2-4)
Complementary angles		⠫⠪⠨⠉⠏	(dots 1-2-4-6, 2-4-6, 4-6, 1-4, 1-2-3-4)
Corresponding angles		⠫⠪⠨⠉	(dots 1-2-4-6, 2-4-6, 4-6, 1-4)
Exterior angles		⠫⠪⠨⠑	(dots 1-2-4-6, 2-4-6, 4-6, 1-5)
Interior angles		⠫⠪⠨⠊	(dots 1-2-4-6, 2-4-6, 4-6, 2-4)

Obtuse angle		⠫⠪⠨⠕	(dots 1-2-4-6, 2-4-6, 4-6, 1-3-5)
Right angle		⠫⠪⠨⠗	(dots 1-2-4-6, 2-4-6, 4-6, 1-2-3-5)
Straight angle		⠫⠪⠨⠎	(dots 1-2-4-6, 2-4-6, 4-6, 2-3-4)
Supplementary angles		⠫⠪⠨⠎⠏	(dots 1-2-4-6, 2-4-6, 4-6, 2-3-4, 1-2-3-4)
Vertical angles		⠫⠪⠨⠧	(dots 1-2-4-6, 2-4-6, 4-6, 1-2-3-6)

Circle

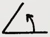
Small circle		⠫⠉⠨⠎	(dots 1-2-4-6, 1-4, 4-6, 2-3-4)

Triangle

Acute triangle		⠫⠞⠨⠁	(dots 1-2-4-6, 2-3-4-5, 4-6, 1)
Isosceles triangle		⠫⠞⠨⠊	(dots 1-2-4-6, 2-3-4-5, 4-6, 2-4)
Obtuse triangle		⠫⠞⠨⠕	(dots 1-2-4-6, 2-3-4-5, 4-6, 1-3-5)
Right triangle		⠫⠞⠨⠗	(dots 1-2-4-6, 2-3-4-5, 4-6, 1-2-3-5)

Shapes with Interior Modification

Angle

Angle with interior arc		⠫⠪⠨⠫⠁	(dots 1-2-4-6, 2-4-6, 4-6, 1-2-4-6, 1)
Angle with interior clockwise arrow		⠫⠪⠨⠫⠪⠢⠔	(dots 1-2-4-6, 2-4-6, 4-6, 1-2-4-6, 2-4-6, 2-6, 3-5)
Angle with interior counterclockwise arrow		⠫⠪⠨⠫⠢⠔⠕	(dots 1-2-4-6, 2-4-6, 4-6, 1-2-4-6, 2-6, 3-5, 1-3-5)

Circle

Circle with interior arrow east		⠫⠉⠨⠫⠒⠒⠕	(dots 1-2-4-6, 1-4, 4-6, 1-2-4-6, 2-5, 2-5, 1-3-5)
Circle with interior arrow west		⠫⠉⠨⠫⠪⠒⠒	(dots 1-2-4-6, 1-4, 4-6, 1-2-4-6, 2-4-6, 2-5, 2-5)

Circle with interior arrow east over interior arrow west	⇄⃝	⠫⠉⠨⠫⠒⠒⠕⠫⠪⠒⠒	(dots 1-2-4-6, 1-4, 4-6, 1-2-4-6, 2-5, 2-5, 1-3-5, 1-2-4-6, 2-4-6, 2-5, 2-5)
Circle with interior arrow west over interior arrow east	⇆⃝	⠫⠉⠨⠫⠪⠒⠒⠫⠒⠒⠕	(dots 1-2-4-6, 1-4, 4-6, 1-2-4-6, 2-4-6, 2-5, 2-5, 1-2-4-6, 2-5, 2-5, 1-3-5)
Circle with interior arrow north	↑⃝	⠫⠉⠨⠫⠣⠒⠒⠕	(dots 1-2-4-6, 1-4, 4-6, 1-2-4-6, 1-2-6, 2-5, 2-5, 1-3-5)
Circle with interior arrow south	↓⃝	⠫⠉⠨⠫⠩⠒⠒⠕	(dots 1-2-4-6, 1-4, 4-6, 1-2-4-6, 1-4-6, 2-5, 2-5, 1-3-5)
Circle with interior arrow north followed by interior arrow south	⇅⃝	⠫⠉⠨⠫⠣⠒⠒⠕⠐⠫⠩⠒⠒⠕	(dots 1-2-4-6, 1-4, 4-6, 1-2-4-6, 1-2-6, 2-5, 2-5, 1-3-5, 5, 1-2-4-6, 1-4-6, 2-5, 2-5, 1-3-5)
Circle with interior arrow south followed by interior arrow north	⇵⃝	⠫⠉⠨⠫⠩⠒⠒⠕⠐⠫⠣⠒⠒⠕	(dots 1-2-4-6, 1-4, 4-6, 1-2-4-6, 1-4-6, 2-5, 2-5, 1-3-5, 5, 1-2-4-6, 1-2-6, 2-5, 2-5, 1-3-5)
Circle with interior capitalized letter	Ⓗ	⠫⠉⠨⠠⠓	(dots 1-2-4-6, 1-4, 4-6, 6, 1-2-5)
Circle with interior cross	⊗	⠫⠉⠨⠈⠡	(dots 1-2-4-6, 1-4, 4-6, 4, 1-6)
Circle with interior dot	⊙	⠫⠉⠨⠡	(dots 1-2-4-6, 1-4, 4-6, 1-6)
Circle with interior minus sign	⊖	⠫⠉⠨⠤	(dots 1-2-4-6, 1-4, 4-6, 3-6)
Circle with interior plus sign	⊕	⠫⠉⠨⠬	(dots 1-2-4-6, 1-4, 4-6, 3-4-6)
Circle with interior horizontal bar between two vertical bars	(|−|)	⠫⠉⠨⠳⠐⠱⠐⠳	(dots 1-2-4-6, 1-4, 4-6, 1-2-5-6, 5, 1-5-6, 5, 1-2-5-6)

Square

Square with interior diagonals	⊠	⠫⠲⠨⠢⠈⠔	(dots 1-2-4-6, 2-5-6, 4-6, 2-6, 4, 3-5)
Square with interior dot	⊡	⠫⠲⠨⠡	(dots 1-2-4-6, 2-5-6, 4-6, 1-6)
Square with interior horizontal bar	⊟	⠫⠲⠨⠱	(dots 1-2-4-6, 2-5-6, 4-6, 1-5-6)
Square with interior vertical bar	◫	⠫⠲⠨⠳	(dots 1-2-4-6, 2-5-6, 4-6, 1-2-5-6)

Square with interior northwest-southeast diagonal	⧅	⠫⠲⠨⠢	(dots 1-2-4-6, 2-5-6, 4-6, 2-6)
Square with interior southwest-northeast diagonal	⧄	⠫⠲⠨⠔	(dots 1-2-4-6, 2-5-6, 4-6, 3-5)

Shapes Modified by Superposition

Angle

Angle with superposed arc	∡	⠫⠪⠈⠫⠁	(dots 1-2-4-6, 2-4-6, 4, 1-2-4-6, 1)

Circle

Circle with superposed horizontal bar	⊖	⠫⠉⠈⠱	(dots 1-2-4-6, 1-4, 4, 1-5-6)
Circle with superposed vertical bar	⦶	⠫⠉⠈⠳	(dots 1-2-4-6, 1-4, 4, 1-2-5-6)

Corners

Northeast corner	┐	⠫⠹	(dots 1-2-4-6, 1-4-5-6)
Northwest corner	┌	⠫⠏	(dots 1-2-4-6, 1-2-3-4)
Southeast corner	┘	⠫⠼	(dots 1-2-4-6, 3-4-5-6)
Southwest corner (Factorial)	└	⠫⠧	(dots 1-2-4-6, 1-2-3-6)

Square

Square with superposed horizontal bar	⊟	⠫⠲⠈⠱	(dots 1-2-4-6, 2-5-6, 4, 1-5-6)
Square with superposed vertical bar	◫	⠫⠲⠈⠳	(dots 1-2-4-6, 2-5-6, 4, 1-2-5-6)

§100. Shape: A shape is a sign which is, in general, a miniature picture or diagram of the object which the sign represents.

A shape is represented by using a letter, numeral, or even a configuration of dots which is suggestive of the shape. The shape indicator must precede the shape symbol. A symbol of shape must be used only for the representation of the corresponding sign of shape; it must never be used to represent the word or phrase which is the name of such a sign of shape.

(1)	⠫⠗	▭	(a rectangle)
(2)	⠫⠉	○	(a circle)
(3)	⠫⠲	□	(a square [4 sides])

(4) ⠫⠖ ⬡ (a regular hexagon [6 sides])

(5) ⠫⠪ ∠ (angle)

(6) ⠫⠒⠒⠕ ⟶ (a right-pointing arrow)

(7) ⠭⠘⠫⠲ $x^{\square}$ (x with a square as superscript)

(8) ⠫⠪ ⠠⠁⠠⠃⠠⠉ ⠨⠅ ⠫⠪ ⠠⠙⠠⠑⠠⠋ ∠ABC = ∠DEF

(angle ABC equals angle DEF)

In some instances, the modifications of shape (see §102-104) are such that it becomes impractical to represent the shape with its modification entirely in braille. In such cases, the corresponding figure should be drawn, and the modification may be shown, either by drawing, or by braille, or by a combination thereof.

§101. **Other Shapes:** Signs of shape which do not appear in the list of Basic Shapes must be represented by the use of one or more alphabetic symbols suggestive of the name of the shape being represented. Care must be exercised not to use an alphabetic symbol to which a meaning is already assigned in the above list. In addition, the transcriber must supply a note of explanation to the reader concerning the name of such a sign of shape. If a combination of alphabetic symbols selected to represent a sign of shape is contractable, the contraction must not be used. The shape indicator must, of course, precede a shape symbol constructed in this way.

(1) ⠫⠍⠝ ☾ mn (moon)

(2) ⠫⠉⠓

ch (church)

a. **Filled-in Shapes:** Any of the closed shapes in the above list (*circle, diamond, ellipse, parallelogram, quadrilateral, rectangle, rhombus, trapezoid, triangle,* or any regular polygon), if they are filled in, must be represented as such by using the boldface-type indicator preceding the shape symbol. The shape indicator must, in turn, precede the boldface-type indicator.

(1) ⠫⠸⠑

or

(filled-in ellipse)

b. **Angle:**

(1) ⠫⠪ ⠠⠁⠠⠃⠠⠉ ∠ABC

(angle ABC)

c. **Circle:**

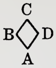
(1) ⠫⠉ ⠠⠗ ○R

(circle R)

d. Diamond:

(1) ⠫⠙ ⠠⠁⠠⠃⠠⠉⠠⠙

(diamond ABCD)



e. Ellipse:

(1) ⠫⠑ ⠠⠗

(ellipse R)

f. Is parallel to:

(1) ⠠⠁⠠⠃ ⠫⠇ ⠠⠉⠠⠙ AB || CD

(AB is parallel to CD)

g. Is perpendicular to:

(1) ⠠⠁⠠⠃ ⠫⠏ ⠠⠁⠠⠉ AB ⊥ AC

(AB is perpendicular to AC)

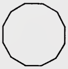
h. Regular Polygons: The list of Basic Shapes contains the shapes for regular polygons up to six sides. Any regular polygon with more than six sides must be represented in the manner suggested, that is, by using the numeral which specifies the number of sides. An irregular polygon, that is, one all of whose sides or all of whose angles are not equal, must *not* be represented in this way. They must be represented as required by the first sentence of the present section. (See §101 and irregular pentagon in Rule XIII.)

(1) ⠫⠦

(regular octagon)

(2) ⠫⠂⠆

(regular polygon of 12 sides)



(3) ⠫⠸⠦

(filled-in regular octagon)

§102. **Shape with Structural Modification:** When a sign, which is a special case of a more general situation, is used, (for example, *right angle* is a special case of *angle*), or when two or more signs of shape are combined to form a composite sign with a more detailed structure, (for example, two *angles* are combined to form *adjacent angles*), the shape which is modified in either of these ways is called a *shape with structural modification.*

The modification is indicated by a letter or combination of letters suggestive of the nature of the modification. The symbol used for indicating the modification must be preceded by the shape-modification indicator, and the combination must directly follow the symbol of shape which is being modified.

Shapes with structural modification not shown in the list of Shapes with Structural Modification must be transcribed in accordance with the principle suggested by those shape symbols. The transcriber must supply a note of explanation to the reader concerning the name of the structural modification. If a combination of alphabetic symbols selected for a structural modifier constitutes a contractable combination, the contraction must not be used.

(1) ⠫⠞⠨⠊

(an isosceles triangle; without modification the shape symbol signifies *triangle*)

(2) ⠫⠪⠨⠚

(adjacent angles; without modification the shape symbol signifies *angle*)

§103. Shape with Interior Modification:

a. When a letter, operation sign, or other sign is placed inside the basic sign of shape, the shape which is modified in this way is called a *shape with interior modification.*

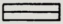
The modification is indicated by using the symbol which corresponds to the modifying sign. This symbol must be preceded by the shape-modification indicator, and the combination must directly follow the symbol of shape which is being modifed.

(1) ⠫⠉⠨⠠⠓

(a circle with an interior Roman capitalized h)

(2) ⠫⠉⠨⠬

(a circle with an interior plus sign)

(3) ⠫⠗⠨⠱ 

(a rectangle with an interior horizontal bar)

b. If two or more interior modifiers, arranged horizontally, occur inside the same basic sign of shape, the corresponding symbols must be separated by the multipurpose indicator, but the shape-modification indicator must be used only once, before the first modifying symbol. The entire combination must directly follow the basic symbol of shape which is being modified.

(1) ⠫⠉⠨⠳⠐⠱⠐⠳

(a circle with an interior vertical bar, followed by a horizontal bar, followed by another vertical bar; the interior modifiers are arranged horizontally and are separated from one another)

c. If two or more interior modifiers, arranged vertically, occur inside the same basic sign of shape, the corresponding symbols must be transcribed successively, without intervening spaces or indicators, beginning with the symbol which corresponds to the uppermost sign and proceeding in descending order. The first modifying symbol must be preceded by the shape-modification indicator, and the entire combination must directly follow the basic symbol of shape which is being modified. None of the interior signs may be regarded as a modifier of any of the others, and the technique for representing modified expressions does not apply.

(1) ⠫⠉⠨⠫⠒⠒⠕⠫⠪⠒⠒

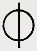
(a circle with an interior right-pointing arrow above a left-pointing arrow)

§104. Shape Modified by Superposition: When a sign is superposed upon a basic sign of shape, the shape which is modified in this way is called a *shape modified by superposition.*

Superposition may be distinguished from interior modification by noting that a superposed sign extends *beyond* the boundary of the basic sign of shape. An interior modifying sign is confined to the inside of the basic sign of shape and extends, at most, *as far as the boundary.*

a. This kind of modification is indicated by using the symbol which corresponds to the superposed sign. This symbol must be preceded by the superposition indicator, and the combination must directly follow the symbol of shape which is being modified.

(1) ⠫⠉⠈⠳



(a circle with a vertical bar through the center extending *beyond* the boundary of the circle)

(2) ⠫⠉⠨⠳

(a circle with a vertical bar through the center extending *as far as* the boundary of the circle)

b. The technique for indicating superposition is not confined to signs of shape. Such a combination is represented by using the symbol corresponding to each of the component signs, and the superposition indicator between the component symbols.

(1) ⠮⠈⠫⠗

(integral sign with superposed rectangle)

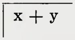
(2) ⠨⠅⠈⠸⠐⠅

(equals sign with superposed inclusion sign)

§105. Corners:

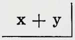
a. The symbols for the corners must not be represented in braille unless some information is superposed. The termination indicator must be used after the expression which is superposed on a corner to indicate the end of the superposition. A corner must not be used for the representation of any of the division format signs.

(1) ⠫⠏⠈⠭⠬⠽⠻



(x plus y superposed on northwest corner)

(2) ⠫⠼⠈⠭⠬⠽⠻

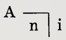


(x plus y superposed on southeast corner)

b. It is preferable to draw the corners and superpose the information spatially.

c. **Northeast corner:** This corner is frequently encountered as a subscript in the mathematics of finance.

(1) ⠠⠁⠰⠫⠹⠈⠝⠻⠊



(northeast corner is subscript, n is superposed on corner, i is outside the corner and to its right; the amount of money for n periods at the rate of i per period)

d. Southwest corner: This corner is sometimes used to indicate the factorial of the mathematical expression which is superposed.

(1) ⠫⠧⠈⠝⠬⠅⠻ $\underline{\left| n + k \right.}$

(n plus k is superposed on southwest corner; the factorial of n plus k)

§106. **Plural of a Sign of Shape:** The plural of a sign of shape is sometimes indicated by placing the letter s on the inside of the sign of shape, rather than to its right. In either case, the plural of the symbol of shape is formed as shown in §15.

(1) ⠫⠪⠸⠄⠎ ⠠⠁⠠⠃⠠⠉ ⠯ ⠠⠙⠠⠑⠠⠋ ∠'s ABC and DEF

(angles ABC and DEF)

(2) ⠫⠞⠸⠄⠎⠸⠲ $\overset{s}{\triangle}$.

(In print, the s is inside the *triangle* shape, while the period is to the right of the *triangle* shape)

RULE XIV—THE USE OF CONTRACTIONS AND SHORT-FORM WORDS

§107. **Use of Contractions and Short-form Words:** In general, the contractions of English Braille must be used, and the rules governing their use must be observed, in transcribing the literary portions of a technical text.

§108. **Non-use of Contractions and Short-form Words:** Contractions and short-form words must not be used:

a. For *to*, *into* and *by* before any mathematical expression and, in particular, they must not be used before a numeric indicator, capitalization indicator, Roman-letter indicator, or italic-type indicator. These indicators have been singled out because the rules of English Braille require the use of these contractions before the corresponding braille symbols in non-technical texts.

(1) ⠠⠋ ⠤⠼⠂⠴ ⠞⠕ ⠬⠂⠴ From —10 to +10

(from minus 10 to plus 10)

(2) ⠠⠊⠋ ⠨⠅ ⠜⠑ ⠙⠊⠧⠊⠙⠫ ⠃⠽ ⠨⠅⠠ ⠮
⠗⠑⠎⠥⠇⠞⠎ ⠜⠑ ⠨⠅⠸⠲

If = are divided by =, the results are =.

(If equals are divided by equals, the results are equal)

(3) ⠠⠙⠑⠉⠕⠍⠏⠕⠎⠑ ⠫⠗ ⠠⠁⠠⠃⠠⠉⠠⠙
⠔⠞⠕ ⠫⠞⠸⠄⠎ ⠠⠁⠠⠃⠠⠉ ⠯ ⠠⠙⠠⠃⠠⠉⠸⠲

Decompose ▭ ABCD into $\overset{s}{\triangle}$ ABC and DBC.

(Decompose rectangle ABCD into triangles ABC and DBC.)

(4) ⠰⠽ ⠊⠎ ⠏⠗⠕⠏⠕⠗⠰⠝⠁⠇ ⠞⠕ ⠇⠕⠛ ⠭

y is proportional to log x

(y is proportional to the logarithm of x)

b. For ordinal endings.

(1) ⠼⠂⠎⠞⠠ ⠼⠆⠝⠙⠠ ⠼⠒⠗⠙⠠ ⠼⠲⠞⠓⠸⠲

1st, 2nd, 3rd, 4th.

(2) ⠊⠞⠓⠠ ⠚⠞⠓⠠ ⠅⠞⠓⠸⠲

ith, jth, kth.

c. In symbols of shape, shape modification, or letters from alphabets other than those for which specific provision is made, in this Code.

d. In abbreviated scientific words or phrases.

(1) ⠎⠊⠝ ⠭ sin x

(sine of x; the *in* contraction must not be used)

(2) ⠉⠕⠎⠓ ⠭ cosh x

(the hyperbolic cosine of x; the *sh* contraction must not be used)

e. In abbreviations of measurement (see §164a).

(1) ⠼⠒ ⠠⠊⠝ 3 In.

(3 inches)

(2) ⠼⠶ ⠍⠊⠸⠌⠍⠊⠝ 7 mi/min

(7 miles per minute)

f. In a word or phrase in the role of a mathematical expression (see §148b for definition) which is in direct contact with a mathematical symbol, which carries a superscript or subscript, which is modified by a sign directly over or directly under it, or which is itself such a superscript, subscript, or modifier.

(1) ⠷⠗⠁⠞⠑⠾ ⠈⠡ ⠷⠞⠊⠍⠑⠾ ⠨⠅ ⠷⠙⠊⠎⠞⠁⠝⠉⠑⠾

(rate) × (time) = (distance)

(each word is enclosed within parentheses)

(2) ⠢⠻⠛⠽ ⠨⠅ ⠍⠁⠎⠎ ⠈⠡ ⠷⠎⠏⠑⠑⠙ ⠷
⠇⠊⠛⠓⠞⠾⠘⠆

(energy = mass × (speed of light)^2)

(3) ⠷⠏⠑⠕⠏⠇⠑ ⠱⠕ ⠞⠗⠁⠧⠑⠇ ⠴⠉⠁⠗⠾ ⠸⠌ ⠷⠂⠴⠴
⠉⠓⠊⠇⠙⠗⠑⠝⠾

(people who travel by car)/(100 children)

(4) ⠼⠒ ⠈⠡ ⠎⠑⠧⠑⠝⠘⠆ ⠬⠲ ⠈⠡ ⠎⠑⠧⠑⠝⠘⠂
⠬⠢ ⠈⠡ ⠎⠑⠧⠑⠝⠘⠴ ⠨⠅ ⠼⠒⠲⠢⠰⠎⠑⠧⠑⠝

$3 \times \text{seven}^2 + 4 \times \text{seven}^1 + 5 \times \text{seven}^0 = 345_{\text{seven}}$

(3 times seven squared plus 4 times seven to the first power plus 5 times seven to the zero power equals 345 to the base seven)

(5) ⠐⠧⠑⠇⠕⠉⠊⠞⠽⠣⠱⠻ $\overline{\text{velocity}}$

(*velocity* with a superscribed bar)

g. When they are likely to be mistaken for mathematical expressions, they must be spelled out in full where the context might cause confusion.

(1) ⠠⠮ ⠓⠽⠏⠑⠮⠎⠊⠎ ⠁⠃⠕⠥⠞ ⠁⠃ ⠊⠎ ⠥⠝⠞⠢⠁⠼⠲

The hypothesis about ab is untenable.

(2) ⠠⠥⠎⠑ ⠞⠓⠑ ⠮ ⠖⠋⠔⠙ ⠮ ⠧⠕⠇⠥⠍⠑⠲

Use the $\int$ to find the volume.

(integral sign after the word *the*)

RULE XV—SIGNS OF OPERATION

Ampersand (and, logical product)	&	⠠⠯	(dots 6, 1-2-3-4-6)
Asterisk (star)	$\ast$	⠈⠼	(dots 4, 3-4-5-6)
Circle with interior dot	$\odot$	⠫⠉⠨⠡	(dots 1-2-4-6, 1-4, 4-6, 1-6)
Circle with interior plus	$\oplus$	⠫⠉⠨⠬	(dots 1-2-4-6, 1-4, 4-6, 3-4-6)
Comma (Use *only* as in §113)	,	⠠⠂	(dots 6, 2)
Division (divided by)	$\div$	⠨⠌	(dots 4-6, 3-4)
Filled-in dot	•	⠸⠡	(dots 4-5-6, 1-6)
Filled-in square	■	⠫⠸⠲	(dots 1-2-4-6, 4-5-6, 2-5-6)

Fraction line (over)			
Complex fraction line	———	⠠⠌	(dots 6, 3-4)
Diagonal fraction line	/	⠸⠌	(dots 4-5-6, 3-4)
Simple fraction line	——	⠌	(dots 3-4)
Hollow dot (degree sign)	°	⠨⠡	(dots 4-6, 1-6)
Hollow square	□	⠫⠲	(dots 1-2-4-6, 2-5-6)
Intersection (cap)	∩	⠨⠩	(dots 4-6, 1-4-6)
Logical product (and, meet)	∧	⠈⠩	(dots 4, 1-4-6)
Logical sum (join, or)	∨	⠈⠬	(dots 4, 3-4-6)
Minus	—	⠤	(dots 3-6)
Minus followed by plus	— +	⠤⠐⠬	(dots 3-6, 5, 3-4-6)
Minus or plus	∓	⠤⠬	(dots 3-6, 3-4-6)
Multiplication (times)			
Cross (cartesian product)	×	⠈⠡	(dots 4, 1-6)
Dot	·	⠡	(dots 1-6)
Number sign; crosshatch; tic-tac-toe	#	⠨⠼	(dots 4-6, 3-4-5-6)
Paragraph mark	¶	⠈⠠⠏	(dots 4, 6, 1-2-3-4)
Plus	+	⠬	(dots 3-4-6)
Plus followed by minus	+ —	⠬⠐⠤	(dots 3-4-6, 5, 3-6)
Plus or minus	±	⠬⠤	(dots 3-4-6, 3-6)
Section mark	§	⠈⠠⠎	(dots 4, 6, 2-3-4)
Union (cup)	∪	⠨⠬	(dots 4-6, 3-4-6)

§109. Ampersand:

(1) ⠠⠁⠠⠯⠠⠃ A & B

(ampersand between A and B)

§110. Asterisk, Paragraph Mark, Section Mark: The asterisk, paragraph mark, and section mark must be represented by the symbols provided for them in this Code; the English Braille symbols must be avoided even when the corresponding signs are used for purposes of reference.

(1) ⠋⠈⠼⠛ f∗g

(asterisk between f and g)

(2) ⠭⠘⠈⠼ x^*

(asterisk is superscript to x)

(3) ⠠⠁⠈⠠⠏⠠⠃ A¶B

(paragraph sign between A and B)

(4) ⠈⠠⠏⠁ ¶a

(paragraph a)

(5) ⠠⠁⠈⠠⠎⠠⠃ A§B

(section mark between A and B)

(6) ⠈⠠⠎⠼⠆ §2

(section 2)

§111. Circle with Interior Dot:

(1) ⠭ ⠫⠉⠨⠡ ⠽ $x \odot y$

(circle with interior dot between x and y)

§112. Circle with Interior Plus:

(1) ⠭ ⠫⠉⠨⠬ ⠽ $x \oplus y$

(circle with interior plus sign between x and y)

§113. Comma: The comma in this list must not be mistaken for the punctuation mark of the same name. The distinction is made by noting that this comma is commonly used in tensor analysis, differential geometry, and related fields, *and* has no space on either side of it; it is used as a sign of operation. (For the comma used as a mark of punctuation, see §18.)

(1) ⠭⠠⠂⠽ x,y

(comma between x and y, no space on either side of the comma)

§114. Division:

(1) ⠼⠂⠆⠨⠌⠒ $12 \div 3$

(12 divided by 3)

§115. Filled-in Dot:

(1) ⠭⠸⠡⠽ $x \bullet y$

(filled-in dot between x and y)

§116. Filled-in Square:

(1) ⠭ ⠫⠸⠲ ⠽ $x \blacksquare y$

(filled-in square between x and y)

(2) ⠼⠔⠰⠶ ⠫⠸⠲ ⠼⠂⠒⠰⠶ $9_7 \blacksquare 13_7$

(filled-in square between 9 sub 7 and 13 sub 7)

§117. Fraction Lines: (For a complete discussion of the rules governing fraction lines, see §29-40.)

§118. Hollow Dot: The hollow dot, when used as a superscript, commonly means *degrees.*

(1) ⠼⠔⠴⠘⠨⠡ 90°

(90 degrees)

(2) ⠋⠨⠡⠛ $f{\circ}g$

(hollow dot between f and g)

§119. Hollow Square: (See §116.)

(1) ⠭ ⠫⠲ ⠽ $x \square y$

(hollow square between x and y)

§120. Intersection, Union: When the intersection sign or the union sign is modified by a superscribed bar, a subscribed bar, or both, the combination is no longer a sign of operation but a sign of comparison compounded vertically (see §133).

These signs are frequently modified below, and are consequently printed wide enough to accommodate the modifier. The variable width of these signs must be ignored in the transcription. Superscripts or subscripts which are sometimes attached to these signs must be treated in the usual manner for handling superscripts and subscripts.

(1) ⠠⠁⠨⠩⠠⠃ A ∩ B

(intersection sign between A and B)

(2) ⠐⠨⠩⠩⠨⠁⠈⠑⠨⠠⠁⠻⠭⠰⠨⠁ $\bigcap\limits_{\alpha\epsilon A} x_\alpha$

(subscribed to the intersection sign is "alpha is an element of capital alpha," to the right of the intersection sign is x sub alpha)

(3) ⠠⠁⠨⠬⠠⠃ A ∪ B

(union sign between A and B)

(4) ⠐⠨⠬⠩⠨⠁ ⠈⠑ ⠨⠠⠁⠻⠭⠰⠨⠁ $\bigcup\limits_{\alpha\epsilon A} x_\alpha$

(subscribed to the union sign is "alpha is an element of capital alpha," to the right of the union sign is x sub alpha)

§121. Logical Product, Logical Sum: When the signs for *logical product* or *logical sum* are modified by a superscribed bar, a subscribed bar, or both, the combination is no longer a sign of operation but a sign of comparison compounded vertically (see §133).

(1) ⠭⠈⠩⠽ $x \wedge y$

(the logical product of x and y)

(2) ⠭⠈⠬⠽ $x \vee y$

(the logical sum of x and y)

§122. Minus:

(1) ⠭⠤⠽ $x - y$

(x minus y)

(2) ⠎⠊⠝ ⠭⠤⠎⠊⠝ ⠽ $\sin x - \sin y$

(the sine of x minus the sine of y)

§123. Minus Followed by Plus: When the signs for *plus* and *minus* are combined either vertically or horizontally, the combination must be regarded as a single sign of operation. Its components must not be divided between braille lines in the transcription.

(1) ⠬⠆⠤⠐⠬⠒ $+2 - +3$

(plus 2, minus a positive 3)

§124. Minus or plus (see §123):

(1) ⠭⠤⠬⠽ $x \mp y$

(x minus or plus y)

§125. Multiplication: The two common multiplication signs, *cross* and *dot,* must not be used interchangeably in the transcription, but the identity of each must be preserved.

a. **Multiplication cross:** The cross is sometimes modified below.

(1) ⠼⠒⠈⠡⠂⠴ 3×10

(3 times 10; cross between 3 and 10)

(2) ⠐⠈⠡⠩⠨⠁⠈⠑⠨⠠⠁⠻⠠⠁⠰⠨⠁ $\underset{\alpha\epsilon A}{\times} A_{\alpha}$

(subscribed to the cross is "alpha is an element of capital alpha," to the right of the cross is capital a sub alpha)

b. **Multiplication Dot:** (See a above.)

(1) ⠭⠡⠽ $x \cdot y$

(x times y; dot between x and y)

§126. Number Sign, Cross-hatch, Tic-tac-toe:

(1) ⠭⠨⠼⠽ $x\#y$

(number sign between x and y)

(2) ⠠⠗⠘⠨⠼ $R^{\#}$

(number sign is superscript to R)

§127. Plus:

(1) ⠭⠬⠽ $x + y$

(x plus y)

(2) ⠎⠊⠝ ⠭ ⠬⠎⠊⠝ ⠽ $\sin x + \sin y$

(the sine of x plus the sign of y)

§128. Plus Followed by Minus (See §123):

(1) ⠤⠼⠒⠬⠐⠤⠢ $-3 + -5$

(minus 3, plus a negative 5)

§129. Plus or Minus (See §123):

(1) ⠭⠬⠤⠽ $x \pm y$

(x plus or minus y)

RULE XVI—COMPARISON SIGNS

Negation Signs			
Oblique (lower left to upper right)	/	⠌	(dots 3-4)
Oblique (upper left to lower right)	\	⠡	(dots 1-6)
Vertical	|	⠳	(dots 1-2-5-6)
Simple Comparison Signs			
Arc			
Opening downward	⌢	⠫⠁	(dots 1-2-4-6, 1)
Opening upward	⌣	⠫⠄	(dots 1-2-4-6, 3)
Arrow			
Left-pointing	⟵	⠫⠪⠒⠒	(dots 1-2-4-6, 2-4-6, 2-5, 2-5)
Right-pointing	⟶	⠫⠒⠒⠕	(dots 1-2-4-6, 2-5, 2-5, 1-3-5)
Two-way	⟷	⠫⠪⠒⠒⠕	(dots 1-2-4-6, 2-4-6, 2-5, 2-5, 1-3-5)
Difference between (excess) (*rare*)	∹	⠫⠒⠒⠅	(dots 1-2-4-6, 2-5, 2-5, 1-3)
Equals (is equals to)			
Ordinary	=	⠨⠅	(dots 4-6, 1-3)
Boldface	**=**	⠸⠨⠅	(dots 4-5-6, 4-6, 1-3)
Greater than (is greater than)			
Ordinary	>	⠨⠂	(dots 4-6, 2)
With curved sides	≻	⠨⠨⠂	(dots 4-6, 4-6, 2)
Rare	⊏	⠨⠉	(dots 4-6, 1-4)
Horizontal bar	—	⠱	(dots 1-5-6)
Identity (is congruent to)	≡	⠸⠇	(dots 4-5-6, 1-2-3)
Inclusion			
Ordinary (is contained in)	⊂	⠸⠐⠅	(dots 4-5-6, 5, 1-3)
Reverse (contains)	⊃	⠸⠨⠂	(dots 4-5-6, 4-6, 2)
Is a Part Of (*rare*)	—∋	⠫⠒⠒⠪	(dots 1-2-4-6, 2-5, 2-5, 2-4-6)

Less than (is less than) **Ordinary**	<	⠐⠅	(dots 5, 1-3)
With curved sides	≺	⠨⠐⠅	(dots 4-6, 5, 1-3)
Rare	⊐	⠨⠤	(dots 4-6, 3-6)
Membership **Ordinary (is an element of)**	ε or ∈ or ∈	⠈⠑	(dots 4, 1-5)
Reverse (contains the element)	϶ or ∋ or ∋	⠈⠢	(dots 4, 2-6)
Pipestrap **Ordinary**	⌢	⠈⠣	(dots 4, 1-2-6)
Reverse	⌣	⠠⠣	(dots 6, 1-2-6)
Proportion (as)	∷	⠰⠆	(dots 5-6, 2-3)
Ratio (is to)	:	⠐⠂	(dots 5, 2)
Tilde			
Simple (is related to)	∼	⠈⠱	(dots 4, 1-5-6)
Extended (is related to)	∿	⠈⠠⠱	(dots 4, 6, 1-5-6)
Variation (varies as)	∝	⠸⠿	(dots 4-5-6, 1-2-3-4-5-6)
Modified Comparison Signs			
Equals Sign			
Caret over	$\overset{\wedge}{=}$	⠐⠨⠅⠣⠸⠣⠻	(dots 5, 4-6, 1-3, 1-2-6, 4-5-6, 1-2-6, 1-2-4-5-6)
Caret under (is projective to, projective correspondence)	$\underset{\wedge}{=}$	⠐⠨⠅⠩⠸⠣⠻	(dots 5, 4-6, 1-3, 1-4-6, 4-5-6, 1-2-6, 1-2-4-5-6)
Degree sign over (is equal in degrees to)	$\overset{\circ}{=}$	⠐⠨⠅⠣⠨⠡⠻	(dots 5, 4-6, 1-3, 1-2-6, 4-6, 1-6, 1-2-4-5-6)
Dot over (is approximately equal to)	$\doteq$	⠐⠨⠅⠣⠡⠻	(dots 5, 4-6, 1-3, 1-2-6, 1-6, 1-2-4-5-6)
Dot over and dot under	$\overset{\cdot}{\underset{\cdot}{=}}$	⠐⠨⠅⠩⠡⠣⠡⠻	(dots 5, 4-6, 1-3, 1-4-6, 1-6, 1-2-6, 1-6, 1-2-4-5-6)
Equilateral triangle over	$\overset{\triangle}{=}$	⠐⠨⠅⠣⠫⠒⠻	(dots 5, 4-6, 1-3, 1-2-6, 1-2-4-6, 2-5, 1-2-4-5-6)

Inverted caret over	$\overset{\vee}{=}$	⠐⠨⠅⠣⠸⠩⠻	(dots 5, 4-6, 1-3, 1-2-6, 4-5-6, 1-4-6, 1-2-4-5-6)
Left-pointing caret over	$\overset{<}{=}$	⠐⠨⠅⠣⠰⠣⠻	(dots 5, 4-6, 1-3, 1-2-6, 5-6, 1-2-6, 1-2-4-5-6)
Question mark over	$\overset{?}{=}$	⠐⠨⠅⠣⠸⠦⠻	(dots 5, 4-6, 1-3, 1-2-6, 4-5-6, 2-3-6, 1-2-4-5-6)
Right-pointing caret over	$\overset{>}{=}$	⠐⠨⠅⠣⠰⠩⠻	(dots 5, 4-6, 1-3, 1-2-6, 5-6, 1-4-6, 1-2-4-5-6)
Two dots over and two dots under	$\underset{\cdot\cdot}{\overset{\cdot\cdot}{=}}$	⠐⠨⠅⠩⠡⠡⠣⠡⠡⠻	(dots 5, 4-6, 1-3, 1-4-6, 1-6, 1-6, 1-2-6, 1-6, 1-6, 1-2-4-5-6)
Vertical bar over	$\overset{\vert}{=}$	⠐⠨⠅⠣⠳⠻	(dots 5, 4-6, 1-3, 1-2-6, 1-2-5-6, 1-2-4-5-6)

Horizontal bar

Caret under (is perspective to, perspective correspondence)	$\underset{\wedge}{-}$	⠐⠱⠩⠸⠣⠻	(dots 5, 1-5-6, 1-4-6, 4-5-6, 1-2-6, 1-2-4-5-6)
Dot over	$\overset{\cdot}{-}$	⠐⠱⠣⠡⠻	(dots 5, 1-5-6, 1-2-6, 1-6, 1-2-4-5-6)

Simple tilde

Dot under	$\underset{\cdot}{\sim}$	⠐⠈⠱⠩⠡⠻	(dots 5, 4, 1-5-6, 1-4-6, 1-6, 1-2-4-5-6)

Comparison Signs Compounded Vertically

Arrow Combinations

Arrow east over arrow west	$\rightleftarrows$	⠫⠒⠒⠕⠫⠪⠒⠒	(dots 1-2-4-6, 2-5, 2-5, 1-3-5, 1-2-4-6, 2-4-6, 2-5, 2-5)
Arrow with upper barb east over arrow with lower barb west	$\rightleftharpoons$	⠫⠒⠒⠈⠕⠫⠠⠪⠒⠒	(dots 1-2-4-6, 2-5, 2-5, 4, 1-3-5, 1-2-4-6, 6, 2-4-6, 2-5, 2-5)
Arrow east over boldface arrow west	$\overset{\rightarrow}{\boldsymbol{\leftarrow}}$	⠫⠒⠒⠕⠫⠸⠪⠒⠒	(dots 1-2-4-6, 2-5, 2-5, 1-3-5, 1-2-4-6, 4-5-6, 2-4-6, 2-5, 2-5)
Arrow west over boldface arrow east	$\overset{\leftarrow}{\boldsymbol{\rightarrow}}$	⠫⠪⠒⠒⠫⠸⠒⠒⠕	(dots 1-2-4-6, 2-4-6, 2-5, 2-5, 1-2-4-6, 4-5-6, 2-5, 2-5, 1-3-5)
Boldface arrow east over arrow west	$\overset{\boldsymbol{\rightarrow}}{\leftarrow}$	⠫⠸⠒⠒⠕⠫⠪⠒⠒	(dots 1-2-4-6, 4-5-6, 2-5, 2-5, 1-3-5, 1-2-4-6, 2-4-6, 2-5, 2-5)
Boldface arrow west over arrow east	$\overset{\boldsymbol{\leftarrow}}{\rightarrow}$	⠫⠸⠪⠒⠒⠫⠒⠒⠕	(dots 1-2-4-6, 4-5-6, 2-4-6, 2-5, 2-5, 1-2-4-6, 2-5, 2-5, 1-3-5)
Boldface arrow east over boldface arrow west	$\overset{\boldsymbol{\rightarrow}}{\boldsymbol{\leftarrow}}$	⠫⠸⠒⠒⠕⠫⠸⠪⠒⠒	(dots 1-2-4-6, 4-5-6, 2-5, 2-5, 1-3-5, 1-2-4-6, 4-5-6, 2-4-6, 2-5, 2-5)

Boldface arrow west over boldface arrow east	$\boldsymbol{\leftrightarrows}$	⠫⠸⠪⠒⠒⠫⠸⠒⠒⠕	(dots 1-2-4-6, 4-5-6, 2-4-6, 2-5, 2-5, 1-2-4-6, 4-5-6, 2-5, 2-5, 1-3-5)
Long arrow east over short arrow west	$\underset{\leftarrow}{\longrightarrow}$	⠫⠒⠒⠒⠕⠫⠪⠒	(dots 1-2-4-6, 2-5, 2-5, 2-5, 1-3-5, 1-2-4-6, 2-4-6, 2-5)
Short arrow east over long arrow west	$\underset{\longleftarrow}{\rightarrow}$	⠫⠒⠕⠫⠪⠒⠒⠒	(dots 1-2-4-6, 2-5, 1-3-5, 1-2-4-6, 2-4-6, 2-5, 2-5, 2-5)
Difference Between (*rare*)	$\bumpeq$	⠈⠣⠱	(dots 4, 1-2-6, 1-5-6)
Equivalence (is equivalent to)	$\asymp$	⠈⠣⠠⠣	(dots 4, 1-2-6, 6, 1-2-6)
Greater than			
Bar over greater than (is equal to or greater than)	$\overline{>}$ or $\eqslantgtr$	⠱⠨⠂	(dots 1-5-6, 4-6, 2)
Bar under greater than (is greater than or equal to)	$\underline{>}$ or $\geqslant$	⠨⠂⠱	(dots 4-6, 2, 1-5-6)
Equals sign over greater than (is equal to or greater than)	$\overset{=}{>}$	⠨⠅⠨⠂	(dots 4-6, 1-3, 4-6, 2)
Equals sign under greater than (is greater than or equal to)	$\underset{=}{>}$	⠨⠂⠨⠅	(dots 4-6, 2, 4-6, 1-3)
Inclusion			
Bar over inclusion (is a subset of)	$\overline{\subset}$	⠱⠸⠐⠅	(dots 1-5-6, 4-5-6, 5, 1-3)
Bar under inclusion (is a subset of)	$\underline{\subset}$	⠸⠐⠅⠱	(dots 4-5-6, 5, 1-3, 1-5-6)
Equals sign over inclusion (is a subset of)	$\overset{=}{\subset}$	⠨⠅⠸⠐⠅	(dots 4-6, 1-3, 4-5-6, 5, 1-3)
Equals sign under inclusion (is a subset of)	$\underset{=}{\subset}$	⠸⠐⠅⠨⠅	(dots 4-5-6, 5, 1-3, 4-6, 1-3)
Intersection (cap)			
Bar under intersection	$\underline{\cap}$	⠨⠩⠱	(dots 4-6, 1-4-6, 1-5-6)
Equals sign under intersection	$\underset{=}{\cap}$	⠨⠩⠨⠅	(dots 4-6, 1-4-6, 4-6, 1-3)
Less than			
Bar over less than (is equal to or less than)	$\overline{<}$ or $\eqslantless$	⠱⠐⠅	(dots 1-5-6, 5, 1-3)

Bar under less than (is less than or equal to)	$\leq$ or $\leqslant$	⠐⠅⠱	(dots 5, 1-3, 1-5-6)
Equals sign over less than (is equal to or less than)	$\overset{=}{<}$	⠨⠅⠐⠅	(dots 4-6, 1-3, 5, 1-3)
Equals sign under less than (is less than or equal to)	$\underset{=}{<}$	⠐⠅⠨⠅	(dots 5, 1-3, 4-6, 1-3)

Logical product (meet)

Bar over logical product	$\overline{\wedge}$	⠱⠈⠩	(dots 1-5-6, 4, 1-4-6)
Bar over and bar under logical product	$\underline{\overline{\wedge}}$	⠱⠈⠩⠱	(dots 1-5-6, 4, 1-4-6, 1-5-6)
Bar over and equals sign under logical product	$\underset{=}{\overline{\wedge}}$	⠱⠈⠩⠨⠅	(dots 1-5-6, 4, 1-4-6, 4-6, 1-3)
Bar under logical product	$\underline{\wedge}$	⠈⠩⠱	(dots 4, 1-4-6, 1-5-6)
Equals sign over logical product	$\overset{=}{\wedge}$	⠨⠅⠈⠩	(dots 4-6, 1-3, 4, 1-4-6)
Equals sign over and bar under logical product	$\underline{\overset{=}{\wedge}}$	⠨⠅⠈⠩⠱	(dots 4-6, 1-3, 4, 1-4-6, 1-5-6)
Equals sign over and equals sign under logical product	$\underset{=}{\overset{=}{\wedge}}$	⠨⠅⠈⠩⠨⠅	(dots 4-6, 1-3, 4, 1-4-6, 4-6, 1-3)
Equals sign under logical product	$\underset{=}{\wedge}$	⠈⠩⠨⠅	(dots 4, 1-4-6, 4-6, 1-3)

Logical Sum (join)

Bar over logical sum	$\overline{\vee}$	⠱⠈⠬	(dots 1-5-6, 4, 3-4-6)
Bar over and bar under logical sum	$\underline{\overline{\vee}}$	⠱⠈⠬⠱	(dots 1-5-6, 4, 3-4-6, 1-5-6)
Bar over and equals sign under logical sum	$\underset{=}{\overline{\vee}}$	⠱⠈⠬⠨⠅	(dots 1-5-6, 4, 3-4-6, 4-6, 1-3)
Bar under logical sum	$\underline{\vee}$	⠈⠬⠱	(dots 4, 3-4-6, 1-5-6)
Equals sign over logical sum	$\overset{=}{\vee}$	⠨⠅⠈⠬	(dots 4-6, 1-3, 4, 3-4-6)
Equals sign over and bar under logical sum	$\underline{\overset{=}{\vee}}$	⠨⠅⠈⠬⠱	(dots 4-6, 1-3, 4, 3-4-6, 1-5-6)
Equals sign over and equals sign under logical sum	$\underset{=}{\overset{=}{\vee}}$	⠨⠅⠈⠬⠨⠅	(dots 4-6, 1-3, 4, 3-4-6, 4-6, 1-3)
Equals sign under logical sum	$\underset{=}{\vee}$	⠈⠬⠨⠅	(dots 4, 3-4-6, 4-6, 1-3)

Reverse inclusion

Bar over reverse inclusion	$\overline{\supset}$	⠱⠸⠨⠂	(dots 1-5-6, 4-5-6, 4-6, 2)
Bar under reverse inclusion	$\underline{\supset}$	⠸⠨⠂⠱	(dots 4-5-6, 4-6, 2, 1-5-6)
Equals sign over reverse inclusion	$\overset{=}{\supset}$	⠨⠅⠸⠨⠂	(dots 4-6, 1-3, 4-5-6, 4-6, 2)
Equals sign under reverse inclusion	$\underset{=}{\supset}$	⠸⠨⠂⠨⠅	(dots 4-5-6, 4-6, 2, 4-6, 1-3)

Tilde (is related to)

Bar over double tilde	$\overline{\approx}$	⠱⠈⠱⠈⠱	(dots 1-5-6, 4, 1-5-6, 4, 1-5-6)
Bar over single tilde	$\overline{\sim}$	⠱⠈⠱	(dots 1-5-6, 4, 1-5-6)
Bar under double tilde	$\underline{\approx}$	⠈⠱⠈⠱⠱	(dots 4, 1-5-6, 4, 1-5-6, 1-5-6)
Bar under single tilde	$\underline{\sim}$	⠈⠱⠱	(dots 4, 1-5-6, 1-5-6)
Double tilde	$\approx$	⠈⠱⠈⠱	(dots 4, 1-5-6, 4, 1-5-6)
Equals sign over double tilde	$\overset{=}{\approx}$	⠨⠅⠈⠱⠈⠱	(dots 4-6, 1-3, 4, 1-5-6, 4, 1-5-6)
Equals sign over single tilde	$\overset{=}{\sim}$	⠨⠅⠈⠱	(dots 4-6, 1-3, 4, 1-5-6)
Equals sign under double tilde	$\underset{=}{\approx}$	⠈⠱⠈⠱⠨⠅	(dots 4, 1-5-6, 4, 1-5-6, 4-6, 1-3)
Equals sign under single tilde	$\underset{=}{\sim}$	⠈⠱⠨⠅	(dots 4, 1-5-6, 4-6, 1-3)

Union (cup)

Bar under union	$\underline{\cup}$	⠨⠬⠱	(dots 4-6, 3-4-6, 1-5-6)
Equals sign under union	$\underset{=}{\cup}$	⠨⠬⠨⠅	(dots 4-6, 3-4-6, 4-6, 1-3)

Comparison Signs Compounded Horizontally

Arrow Combinations

Arrow north followed by arrow south	$\uparrow\downarrow$	⠫⠣⠒⠒⠕⠐⠫⠩⠒⠒⠕	(dots 1-2-4-6, 1-2-6, 2-5, 2-5, 1-3-5, 5, 1-2-4-6, 1-4-6, 2-5, 2-5, 1-3-5)
Arrow south followed by arrow north	$\downarrow\uparrow$	⠫⠩⠒⠒⠕⠐⠫⠣⠒⠒⠕	(dots 1-2-4-6, 1-4-6, 2-5, 2-5, 1-3-5, 5, 1-2-4-6, 1-2-6, 2-5, 2-5, 1-3-5)

Arrow north followed by boldface arrow south	↑**↓**	⠫⠣⠒⠒⠕⠐⠫⠩⠸⠒⠒⠕	(dots 1-2-4-6, 1-2-6, 2-5, 2-5, 1-3-5, 5, 1-2-4-6, 1-4-6, 4-5-6, 2-5, 2-5, 1-3-5)
Arrow south followed by boldface arrow north	↓**↑**	⠫⠩⠒⠒⠕⠐⠫⠣⠸⠒⠒⠕	(dots 1-2-4-6, 1-4-6, 2-5, 2-5, 1-3-5, 5, 1-2-4-6, 1-2-6, 4-5-6, 2-5, 2-5, 1-3-5)
Boldface arrow north followed by arrow south	**↑**↓	⠫⠣⠸⠒⠒⠕⠐⠫⠩⠒⠒⠕	(dots 1-2-4-6, 1-2-6, 4-5-6, 2-5, 2-5, 1-3-5, 5, 1-2-4-6, 1-4-6, 2-5, 2-5, 1-3-5)
Boldface arrow south followed by arrow north	**↓**↑	⠫⠩⠸⠒⠒⠕⠐⠫⠣⠒⠒⠕	(dots 1-2-4-6, 1-4-6, 4-5-6, 2-5, 2-5, 1-3-5, 5, 1-2-4-6, 1-2-6, 2-5, 2-5, 1-3-5)
Boldface arrow north followed by boldface arrow south	**↑↓**	⠫⠣⠸⠒⠒⠕⠐⠫⠩⠸⠒⠒⠕	(dots 1-2-4-6, 1-2-6, 4-5-6, 2-5, 2-5, 1-3-5, 5, 1-2-4-6, 1-4-6, 4-5-6, 2-5, 2-5, 1-3-5)
Boldface arrow south followed by boldface arrow north	**↓↑**	⠫⠩⠸⠒⠒⠕⠐⠫⠣⠸⠒⠒⠕	(dots 1-2-4-6, 1-4-6, 4-5-6, 2-5, 2-5, 1-3-5, 5, 1-2-4-6, 1-2-6, 4-5-6, 2-5, 2-5, 1-3-5)

Greater Than

Followed by less than	$> <$	⠨⠂⠐⠐⠅	(dots 4-6, 2, 5, 5, 1-3)
Followed by equals sign followed by less than	$> = <$	⠨⠂⠐⠨⠅⠐⠐⠅	(dots 4-6, 2, 5, 4-6, 1-3, 5, 5, 1-3)

Less Than

Followed by greater than	$< >$	⠐⠅⠐⠨⠂	(dots 5, 1-3, 5, 4-6, 2)
Followed by equals sign followed by greater than	$< = >$	⠐⠅⠐⠨⠅⠐⠨⠂	(dots 5, 1-3, 5, 4-6, 1-3, 5, 4-6, 2)
Spear West-East followed by Identity Sign, followed by Tilde	$\Leftrightarrow \equiv \sim$	⠫⠪⠶⠶⠕⠐⠸⠇⠐⠈⠱	(dots 1-2-4-6, 2-4-6, 2-3-5-6, 2-3-5-6, 1-3-5, 5, 4-5-6, 1-2-3, 5, 4, 1-5-6)

Comparison Signs Compounded by Superposition

Dot

Between bars of equals sign	$\overset{-}{\underset{-}{\cdot}}$	⠡⠈⠨⠅	(dots 1-6, 4, 4-6, 1-3)
Within inclusion sign	⪽	⠡⠈⠸⠐⠅	(dots 1-6, 4, 4-5-6, 5, 1-3)
Within reverse inclusion sign	⪾	⠡⠈⠸⠨⠂	(dots 1-6, 4, 4-5-6, 4-6, 2)

Equals sign			
Through inclusion sign	⋹	⠨⠅⠈⠸⠐⠅	(dots 4-6, 1-3, 4, 4-5-6, 5, 1-3)
Through reverse inclusion sign		⠨⠅⠈⠸⠨⠂	(dots 4-6, 1-3, 4, 4-5-6, 4-6, 2)
Greater than			
Nest of two with straight sides (is large compared with)	≫	⠨⠂⠈⠨⠂	(dots 4-6, 2, 4, 4-6, 2)
Nest of two with curved sides	⪼	⠨⠨⠂⠈⠨⠨⠂	(dots 4-6, 4-6, 2, 4, 4-6, 4-6, 2)
Horizontal Bar			
Through inclusion sign	⋲	⠱⠈⠸⠐⠅	(dots 1-5-6, 4, 4-5-6, 5, 1-3)
Through reverse inclusion sign	⋺	⠱⠈⠸⠨⠂	(dots 1-5-6, 4, 4-5-6, 4-6, 2)
Interlocking Parentheses	≬	⠾⠈⠈⠷	(dots 2-3-4-5-6, 4, 4, 1-2-3-5-6)
Left Parenthesis on Counterclockwise Arrow Shaft		⠷⠈⠫⠢⠔⠕	(dots 1-2-3-5-6, 4, 1-2-4-6, 2-6, 3-5, 1-3-5)
Less than			
Nest of two with straight sides (is small compared with)	≪	⠐⠅⠈⠐⠅	(dots 5, 1-3, 4, 5, 1-3)
Nest of two with curved sides	⪻	⠨⠐⠅⠈⠨⠐⠅	(dots 4-6, 5, 1-3, 4, 4-6, 5, 1-3)
Vertical Bar			
Through shaft of arrow east	⇸	⠳⠈⠫⠒⠒⠕	(dots 1-2-5-6, 4, 1-2-4-6, 2-5, 2-5, 1-3-5)
Through shaft of arrow west	⇷	⠳⠈⠫⠪⠒⠒	(dots 1-2-5-6, 4, 1-2-4-6, 2-4-6, 2-5, 2-5)

§130. **Negation Signs:** While the negation signs are not, of themselves, comparison signs, they are used generally in conjunction with comparison signs. The oblique negation signs must not be mistaken for the cancellation signs. The distinction is made by noting that the cancellation sign is never applied to a comparison sign. The negation sign, on the other hand, is never applied to a mathematical expression containing letters or numerals.

In transcribing a comparison sign, the negation sign, if present, must be represented first. The negation symbol must never be separated from its associated comparison symbol by transition to another braille line.

(1) ⠌⠨⠅ $\neq$

(is not equal to; oblique negation sign in print)

(2) ⠳⠨⠅ ⧧

(is not equal to; vertical negation sign in print)

(3) ⠡⠈⠑ $\notin$

(is not an element of; oblique negation sign in print, from upper left to lower right)

(4) ⠌⠫⠇ $\nparallel$

(is not parallel to; oblique negation sign in print, from lower left to upper right)

(5) ⠡⠐⠅ $\nless$

(is not less than; oblique negation sign in print, from upper left to lower right)

(6) ⠳⠨⠂⠨⠅ $\ngeqq$

(is not greater than or equal to; vertical negation sign in print)

§131. Simple Comparison Signs:

a. Arc opening downward:

(1) ⠭ ⠫⠁ ⠽ $x \frown y$

(arc opening downward between x and y)

b. Arc opening upward:

(1) ⠭ ⠫⠄ ⠽ $x \smile y$

(arc opening upward between x and y)

c. Arrows: A detailed discussion of the construction of arrows of many types is presented in §136-141. The arrows in the list of simple comparison signs are those which occur with the greatest frequency.

A left-pointing arrow is commonly used with the meanings *comes from* and *is implied by.*

A right-pointing arrow is commonly used with the meanings *approaches, corresponds to, implies, maps into,* and *yields.*

A two-way arrow is commonly used with the meanings *if and only if, one-to-one mapping,* and *reversible reaction.*

(1) ⠠⠃ ⠫⠪⠒⠒ ⠠⠁ $B \leftarrow A$

(left-pointing arrow between B and A)

(2) ⠠⠁ ⠫⠒⠒⠕ ⠠⠃ $A \rightarrow B$

(right-pointing arrow between A and B)

(3) ⠠⠁ ⠫⠪⠒⠒⠕ ⠠⠃ $A \leftrightarrow B$

(two-way arrow between A and B)

d. Equals:

(1) ⠭ ⠨⠅ ⠽ $x = y$

(x equals y)

(2) ⠐⠨⠠⠎⠩⠝⠨⠅⠴⠣⠠⠿⠻⠁⠰⠝ $\sum_{n=0}^{\infty} a_n$

(directly under the Greek capitalized sigma is n equals 0, directly over the Greek capitalized sigma is the infinity sign, directly to the right of the Greek capitalized sigma is a sub n)

e. Boldface equals sign:

(1) ⠷⠁⠠ ⠃⠾ ⠸⠨⠅ ⠷⠉⠠ ⠙⠾ ⠊⠋ ⠯ ⠕⠝⠇⠽
⠊⠋ ⠁ ⠨⠅ ⠉ ⠯ ⠃ ⠨⠅ ⠙

$(a, b) \boldsymbol{=} (c, d)$ if and only if $a = c$ and $b = d$

(boldface equals sign between the pairs of letters enclosed in parentheses, the other two equals signs are ordinary)

f. Greater than:

(1) ⠭ ⠨⠂ ⠽ $x > y$

(x is greater than y)

g. Horizontal bar: The horizontal bar is not used by itself as a simple comparison sign. When it occurs with a dot or a caret directly over or directly under it, the combination is a modified sign of comparison (see §132). When it occurs directly over or directly under another simple comparison sign, the combination is a comparison sign compounded vertically (see §133). In this latter connection, the horizontal bar is often used in place of, and with the meaning of, *equals.*

h. Identity: This sign most generally has the meanings of *is congruent to, is identical with,* and *is identically equal to.* This symbol must not be used for *is congruent to* in geometry, if another sign is employed for this purpose in print.

(1) ⠋⠷⠭⠾ ⠸⠇ ⠼⠴ $f(x) \equiv 0$

(f of x is identically equal to 0)

(2) ⠼⠆ ⠸⠇ ⠼⠢ ⠷⠍⠕⠙ ⠒⠾ $2 \equiv 5 \ (\mathrm{mod}\ 3)$

(2 is congruent to 5 modulo 3)

i. Inclusion: This sign is often read as *is contained in* or *is a subset of.*

(1) ⠠⠭ ⠸⠐⠅ ⠠⠽ $X \subset Y$

(X is a subset of Y)

(2) ⠐⠨⠬⠩⠠⠁ ⠸⠐⠅ ⠠⠋⠻⠠⠁ $\bigcup\limits_{A \subset F} A$

(directly under the union sign is A is a subset of F, to the right of the union sign is A; for spacing see §176a)

j. **Reverse inclusion:** This sign is often read as *contains.*

(1) ⠠⠽ ⠸⠨⠂ ⠠⠭ $Y \supset X$

(Y contains X)

k. **Less than:**

(1) ⠭ ⠐⠅ ⠽ $x < y$

(x is less than y)

(2) ⠐⠨⠠⠎⠩⠊⠐⠅⠚⠻⠁⠰⠊⠚ $\sum\limits_{i<j} a_{ij}$

(directly under the Greek capitalized sigma is i is less than j, to the right of the Greek capitalized sigma is a sub ij; for spacing, see §176a)

l. **Membership:** This sign is usually read as *belongs to, is a member of,* or *is an element of.* This sign must not be mistaken for the Greek lower case epsilon, even though it is sometimes referred to by that very name. This sign is generally used when speaking about sets and the elements of which they are composed. When the Greek lower case epsilon is used in the same textbook, the publisher usually makes a sufficient distinction between the two signs to prevent this confusion.

(1) ⠭ ⠈⠑ ⠠⠁ $x \in A$

(x is an element of A)

m. **Reverse membership:**

(1) ⠠⠁ ⠈⠢ ⠭ $A \ni x$

(A contains the element x)

n. **Pipestrap:**

(1) ⠠⠁ ⠈⠣ ⠠⠃ $A \frown B$

(pipestrap between A and B)

o. **Reverse pipestrap:**

(1) ⠠⠁ ⠠⠣ ⠠⠃ $A \smile B$

(reverse pipestrap between A and B)

p. **Proportion:** This sign is generally read as *as.*

(1) ⠼⠂ ⠐⠂ ⠼⠆ ⠰⠆ ⠼⠒ ⠐⠂ ⠼⠖ $1 : 2 :: 3 : 6$

(1 is to 2 as 3 is to 6)

q. **Ratio:** This sign is generally read as *is to.*

(1) ⠁⠬⠃ ⠐⠂ ⠃ ⠰⠆ ⠉⠬⠙ ⠐⠂ ⠙ $a + b : b :: c + d : d$

(a plus b is to b as c plus d is to d)

r. **Tilde:** When the tilde, simple or extended, occurs with a dot or caret directly over or directly under it, the combination is a modified sign of comparison (see §132). When it occurs directly over or directly under another simple comparison sign, the combination is a comparison sign compounded vertically (see §133).

s. **Extended tilde** (See **r**)**:** This sign is often read as *is related to.*

(1) ⠭ ⠈⠠⠱ ⠽ $x \sim\!\!\sim y$

(extended tilde between x and y)

t. **Simple tilde** (See r)**:** This sign is often read as *is equivalent to, is related to,* or *is similar to.*

(1) ⠭ ⠈⠱ ⠽ $x \sim y$

(simple tilde between x and y)

u. **Variation:** This sign is usually read as *varies as.*

(1) ⠭ ⠸⠿ ⠽ $x \propto y$

(x varies as y)

§132. **Modified Comparison Signs:** The modified comparison signs in the above list are those which have been encountered in mathematical and scientific literature. Their construction is in accordance with the rules for the representation of modified expressions (see §77-84). Modified signs of comparison other than those in the above list must be constructed in accordance with the same principle. The components of a modified comparison symbol must never be separated by transition to another braille line.

§133. **Comparison Signs Compounded Vertically:** The transcriber must represent a vertical arrangement of simple comparison signs as a horizontal succession of the corresponding simple comparison symbols without intervening spaces, the first symbol corresponding to the uppermost sign. The components of a comparison sign must never be separated by transition to another braille line.

The braille reader must interpret a succession of simple comparison symbols without intervening spaces as representing the fact that the corresponding signs are arranged vertically in print, in descending order, the uppermost sign corresponding to the first symbol.

Comparison signs compounded vertically not shown in the list must be transcribed in accordance with the above principles.

a. **Arrow combinations:** These signs are used primarily in chemistry to convey the idea that a chemical reaction, although reversible, takes place more easily in one direction than in the other direction.

b. Intersection, Union: The intersection sign and the union sign, when unmodified, are not simple comparison signs, but operation signs (see §120).

c. Bar over single tilde: When a vertical bar occurs through this sign, the combination sometimes is read as *is divisible by.*

(1) ⠳⠱⠈⠱

(is divisible by)

d. Double tilde: When a diagonal line occurs through this sign, the combination is sometimes read as *of the form of.*

(1) ⠌⠈⠱⠈⠱ $\not\approx$

(of the form of)

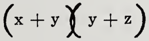
e. Logical Product, Logical Sum: The logical product sign and the logical sum sign, when unmodified, are not simple comparison signs but signs of operation (see §121).

§134. **Comparison Signs Compounded Horizontally:** The transcriber must represent a horizontal succession of comparison signs by placing the multipurpose indicator between the corresponding comparison symbols. The components of a comparison sign compounded horizontally must never be separated by transition to another braille line. Comparison signs compounded horizontally, which are not shown in the list, must be transcribed in accordance with the above principle. The arrow combinations in the above list are used principally in chemistry.

§135. **Comparison Signs Compounded by Superposition:** In a comparison sign compounded by superposition, it is not required that both components be simple comparison signs; it is sufficient that one of the components be such a sign (see §104b). In representing a comparison sign compounded by superposition, the symbol which corresponds to the smaller or simpler sign must be indicated first, and the superposition indicator must be placed between the component symbols. The components of a comparison sign compounded by superposition must never be separated by transition to another braille line.

a. Nest of Greater Than Signs or Less Than Signs: A nest of greater than or less than signs, whether with curved or straight sides, may contain more than two components. In such a situation, the same number of component symbols must be transcribed as occur in the corresponding sign, and the superposition indicator must be placed between the individual components. (See §104b.)

b. Interlocking Parentheses:

(1) ⠷⠭⠬⠽⠾⠈⠈⠷⠽⠬⠵⠾ 

(the outer parentheses are normal, but the interior parentheses are interlocking)

RULE XVII—REPRESENTATION OF ARROWS

Arrow Components

Arrow Directions

Depresses nearer arrowhead by 45 degrees	⠰	(dots 5-6)
Elevates nearer arrowhead by 45 degrees	⠘	(dots 4-5)

Makes nearer arrowhead point north		⠣	(dots 1-2-6)
Makes nearer arrowhead point south		⠩	(dots 1-4-6)
Arrow shafts			
Curved	(or)	⠢⠔	(dots 2-6, 3-5)
Long double	══	⠶⠶⠶	(dots 2-3-5-6, 2-3-5-6, 2-3-5-6)
Long single	——	⠒⠒⠒	(dots 2-5, 2-5, 2-5)
Ordinary double	═	⠶⠶	(dots 2-3-5-6, 2-3-5-6)
Ordinary single	—	⠒⠒	(dots 2-5, 2-5)
Short double	=	⠶	(dots 2-3-5-6)
Short single	–	⠒	(dots 2-5)
Arrow types			
Boldface		⠸	(dots 4-5-6)
Ordinary (no indicator)			
Arrowheads			
Barbed left full	<	⠪	(dots 2-4-6)
Barbed left lower	\	⠠⠪	(dots 6, 2-4-6)
Barbed left upper	/	⠈⠪	(dots 4, 2-4-6)
Barbed right full	>	⠕	(dots 1-3-5)
Barbed right lower	/	⠠⠕	(dots 6, 1-3-5)
Barbed right upper	\	⠈⠕	(dots 4, 1-3-5)
Blunted left full	[	⠿	(dots 1-2-3-4-5-6)
Blunted left lower	⌊	⠠⠿	(dots 6, 1-2-3-4-5-6)
Blunted left upper	⌈	⠈⠿	(dots 4, 1-2-3-4-5-6)

Blunted right full	]	⠿	(dots 1-2-3-4-5-6)
Blunted right lower	⌋	⠠⠿	(dots 6, 1-2-3-4-5-6)
Blunted right upper	⌉	⠈⠿	(dots 4, 1-2-3-4-5-6)
Curved left full	(	⠯	(dots 1-2-3-4-6)
Curved left lower	⎝	⠠⠯	(dots 6, 1-2-3-4-6)
Curved left upper	⎛	⠈⠯	(dots 4, 1-2-3-4-6)
Curved right full	)	⠽	(dots 1-3-4-5-6)
Curved right lower	⎠	⠠⠽	(dots 6, 1-3-4-5-6)
Curved right upper	⎞	⠈⠽	(dots 4, 1-3-4-5-6)
Straight left full	├	⠳	(dots 1-2-5-6)
Straight left lower	⌈	⠠⠳	(dots 6, 1-2-5-6)
Straight left upper	⌊	⠈⠳	(dots 4, 1-2-5-6)
Straight right full	┤	⠳	(dots 1-2-5-6)
Straight right lower	⌉	⠠⠳	(dots 6, 1-2-5-6)
Straight right upper	⌋	⠈⠳	(dots 4, 1-2-5-6)

§136. **Arrow Components:** There is a large class of signs in the form of arrows which differ from each other in several ways. In the above list, there is presented an assortment of arrow components from which such signs are constructed. The entire construction is a simple comparison symbol.

§137. **Six-step Rule for Construction of Arrows:**

a. The *shape indicator* must introduce every arrow symbol.

b. The *arrow direction* must be indicated next (see §138).

c. The *arrow type* must be indicated next (see §140).

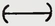
d. The *left arrowhead,* if any, must be indicated next (see §141).

e. The *arrow shaft* must be indicated next (see §139).

f. The *right arrowhead,* if any, must be indicated last (see §141).

(1) ⠫⠣⠸⠪⠒⠒⠕ **↕**

(arrow, north-south, boldface, barbed arrowheads at both ends)

(2) ⠫⠯⠒⠒⠽ 

(arrow, west-east, ordinary type, curved arrowheads at both ends)

(3) ⠫⠘⠿⠶⠶

(spear, northwest, blunted arrowhead)

§138. **Arrow Directions:** It is possible to represent eight arrow directions by making proper use of the direction indicators.

a. The two horizontal directions, *east* and *west*, require no indicator.

(1) ⠫⠒⠒⠕ →

(arrow east)

(2) ⠫⠪⠒⠒ ←

(arrow west)

(3) ⠫⠪⠒⠒⠕ ↔

(arrow west-east)

b. The two vertical directions, *north* and *south*, require the *directly over* indicator or the *directly under* indicator, respectively. The *directly over* indicator "makes the arrowhead point *north*"; the *directly under* indicator "makes the arrowhead point *south*". If a vertical arrow is printed with one arrowhead, it must be transcribed by using the proper symbol for a *right* arrowhead, and not a left one.

(1) ⠫⠣⠒⠒⠕ ↑

(arrow north)

(2) ⠫⠩⠒⠒⠕ ↓

(arrow south)

(3) ⠫⠣⠪⠒⠒⠕ ↕

(arrow north-south)

c. The four oblique directions, *northeast*, *northwest*, *southeast*, and *southwest*, require the superscript indicator or the subscript indicator. The superscript indicator "elevates the nearer arrowhead (if there are two) by 45 degrees from the horizontal position"; the subscript indicator "depresses the nearer arrowhead (if there are two) by 45 degrees from the horizontal position".

(1) ⠫⠘⠒⠒⠕ ↗

(arrow, northeast)

(2) ⠫⠘⠪⠒⠒ ↖
(arrow, northwest)

(3) ⠫⠰⠒⠒⠕ ↘
(arrow, southeast)

(4) ⠫⠰⠪⠒⠒ ↙
(arrow, southwest)

(5) ⠫⠘⠪⠒⠒⠕ ⤡
(arrow, northwest-southeast)

(6) ⠫⠰⠪⠒⠒⠕ ⤢
(arrow, southwest-northeast)

§139. **Arrow Shafts:** An arrow shaft may be curved or straight, single or double, long or short.

a. If an arrow shaft is curved, the direction of curvature is indicated by a left arrowhead or a right arrowhead. A curved arrow shaft followed by a right arrowhead represents a counterclockwise arrow; a curved arrow shaft preceded by a left arrowhead represents a clockwise arrow.

(1) ⠫⠢⠔⠕ ↺
(arrow, counterclockwise)

(2) ⠫⠪⠢⠔ ↻
(arrow, clockwise)

b. Most arrow shafts are single. An arrow with a double arrow shaft is sometimes called a *spear.*

(1) ⠫⠶⠶⠕ ⟹
(spear, east)

(2) ⠫⠪⠶⠶ ⟸
(spear, west)

(3) ⠫⠪⠶⠶⠕ ⟺
(spear, west-east)

c. The length of an arrow shaft is indicated by the number of repetitions of the braille arrow-shaft symbol. The list distinguishes three lengths, but other lengths may be indicated by repeating the braille arrow-shaft symbol a suitable number of times.

(1) ⠫⠒⠕ →
(short arrow, east)

(2) ⠫⠪⠒ ←

(short arrow, west)

(3) ⠫⠪⠒⠕ ↔

(short arrow, west-east)

(4) ⠫⠒⠒⠒⠕ ⟶

(long arrow, east)

(5) ⠫⠪⠒⠒⠒ ⟵

(long arrow, west)

(6) ⠫⠪⠒⠒⠒⠕ ⟷

(long arrow, west-east)

§140. Arrow Types: Most arrows are printed in ordinary type. In that case, no indicator is required. If an arrow is printed in boldface type, the boldface type indicator is required.

(1) ⠫⠸⠒⠒⠕ **⟶**

(boldface arrow, east)

(2) ⠫⠸⠪⠒⠒ **⟵**

(boldface arrow, west)

(3) ⠫⠸⠪⠒⠒⠕ **⟷**

(boldface arrow, west-east)

§141. Arrowheads:

a. Most arrowheads are barbed. However, arrowheads also occur as blunted, curved, or straight. They may occur at the left end, right end, or at both ends, of the arrow shaft.

(1) ⠫⠒⠒⠿

(arrow, east; blunted arrowhead)

(2) ⠫⠿⠒⠒

(arrow, west; blunted arrowhead)

(3) ⠫⠿⠒⠒⠿

(arrow, west-east; blunted arrowheads)

(4) ⠫⠒⠒⠽

(arrow, east; curved arrowhead)

(5) ⠫⠯⠒⠒

(arrow, west; curved arrowhead)

(6) ⠫⠯⠒⠒⠽

(arrow, west-east; curved arrowheads)

(7) ⠫⠒⠒⠳

(arrow, east; straight arrowhead)

(8) ⠫⠳⠒⠒

(arrow, west; straight arrowhead)

(9) ⠫⠳⠒⠒⠳

(arrow, west-east; straight arrowheads)

b. An arrowhead with its upper half only, or its lower half only, may also be present. Any combination of arrowheads — barbed, blunted, curved, or straight, left or right, full, lower half, or upper half, may occur.

(1) ⠫⠈⠪⠒⠒

(arrow, left upper barb only)

(2) ⠫⠠⠪⠒⠒

(arrow, left lower barb only)

(3) ⠫⠒⠒⠈⠕

(arrow, right upper barb only)

(4) ⠫⠒⠒⠠⠕

(arrow, right lower barb only)

(5) ⠫⠈⠪⠒⠒⠈⠕

(arrow, upper barbs only)

(6) ⠫⠠⠪⠒⠒⠠⠕

(arrow, lower barbs only)

(7) ⠫⠈⠪⠒⠒⠠⠕

(arrow, left upper barb and right lower barb)

(8) ⠫⠠⠪⠒⠒⠈⠕

(arrow, left lower barb and right upper barb)

(9) ⠫⠈⠪⠒⠒⠕

(arrow, left upper barb and full right barb)

(10) ⠫⠠⠪⠒⠒⠕

(arrow, left lower barb and full right barb)

(11) ⠫⠪⠒⠒⠈⠕

(arrow, full left barb and right upper barb)

(12) ⠫⠪⠒⠒⠠⠕

(arrow, full left barb and right lower barb)

(13) ⠫⠪⠒⠒⠕

(arrow, full left and right barbs)

RULE XVIII—GROUPING SIGNS

Angle Brackets (angular parentheses)			
Left	⟨	⠨⠨⠷	(dots 4-6, 4-6, 1-2-3-5-6)
Right	⟩	⠨⠨⠾	(dots 4-6, 4-6, 2-3-4-5-6)
Barred Braces			
Left	⦃	⠨⠸⠷	(dots 4-6, 4-5-6, 1-2-3-5-6)
Right	⦄	⠨⠸⠾	(dots 4-6, 4-5-6, 2-3-4-5-6)
Barred Brackets			
Left	⟦	⠈⠸⠷	(dots 4, 4-5-6, 1-2-3-5-6)
Right	⟧	⠈⠸⠾	(dots 4, 4-5-6, 2-3-4-5-6)
Braces (curly brackets)			
Left	{	⠨⠷	(dots 4-6, 1-2-3-5-6)
Right	}	⠨⠾	(dots 4-6, 2-3-4-5-6)

Brackets (square brackets)

Left	[	⠈⠷	(dots 4, 1-2-3-5-6)
Right	]	⠈⠾	(dots 4, 2-3-4-5-6)
Left boldface	**[**	⠸⠈⠷	(dots 4-5-6, 4, 1-2-3-5-6)
Right boldface	**]**	⠸⠈⠾	(dots 4-5-6, 4, 2-3-4-5-6)

Parentheses (round brackets)

Left	(	⠷	(dots 1-2-3-5-6)
Right	)	⠾	(dots 2-3-4-5-6)

Vertical Bar

Single	|	⠳	(dots 1-2-5-6)
Double	||	⠳⠳	(dots 1-2-5-6, 1-2-5-6)
Single boldface	**|**	⠸⠳	(dots 4-5-6, 1-2-5-6)
Double boldface	**||**	⠸⠳⠸⠳	(dots 4-5-6, 1-2-5-6, 4-5-6, 1-2-5-6)

§142. **Signs of Grouping:** Although signs of grouping most commonly occur in pairs, this is not always so. If an opening grouping sign occurs without being followed later by the corresponding closing sign, or if the closing grouping sign occurs without having been preceded by the corresponding opening sign, this situation must be preserved in the transcription.

§143. **Enlarged Grouping Signs** (see §98-99).

§144. **Angle Brackets:** These must not be mistaken for the *less than* and *greater than* signs, respectively. The distinction is made by noting that the angle of opening between the sides of the angle brackets sign is considerably larger than the angle of opening between the sides of the *less than* and *greater than* signs.

§145. **Horizontal Grouping Signs:** When a horizontal grouping sign occurs over or under a mathematical expression, it must be regarded as a modifier, and the combination transcribed according to the rules in §78. The left, rather than the right, grouping symbol must be used in this situation. However, when it seem advisable, for any reason, to do so, the grouping signs may be drawn in.

(1) ⠐⠭⠬⠽⠣⠨⠷⠻ $\overbrace{x + y}$

(horizontal brace over x plus y)

(2) ⠐⠭⠬⠽⠩⠨⠷⠻ $\underbrace{x + y}$

(horizontal brace under x plus y)

(3) ⠐⠭⠬⠽⠣⠈⠷⠻ $\overbracket{x + y}$

(horizontal bracket over x plus y)

(4) ⠐⠭⠬⠽⠩⠈⠷⠻ $\underbracket{x + y}$

(horizontal bracket under x plus y)

§146. **Boldface Brackets:** Boldface brackets are often used to designate the *integer function.*

(1) ⠸⠈⠷⠭⠸⠈⠾ $\boldsymbol{[}x\boldsymbol{]}$

(boldface brackets enclose x)

§147. **Mathematical Grouping Symbols:** The parentheses, brackets, and braces in the above list are those which are used in mathematics and are referred to as *mathematical parentheses, mathematical brackets*, and *mathematical braces.* By contrast, the parentheses, braces, and brackets of §24 are punctuation marks and are referred to as *literary parentheses, literary braces* and *literary brackets.*

§148. **Use of Mathematical Grouping Symbols:** Mathematical grouping symbols must be used:

a. In association with expressions which are entirely mathematical. This includes single letters, Arabic and Roman numerals, as well as longer mathematical expressions. Numerals and letters which are enclosed may be within the body of the text, or they may be used to identify formulas, the steps of an outline, or similar items. Arabic numerals so enclosed do not require the numeric indicator; Roman letters so enclosed do not require the Roman-letter indicator.

(1) ⠷⠭⠬⠽⠾ $(x + y)$

(x plus y enclosed within parentheses)

(2) ⠈⠷⠭⠬⠽⠈⠾ $[x + y]$

(x plus y enclosed within brackets)

(3) ⠈⠷⠠⠋ ⠨⠅ ⠼⠂⠨⠦⠠⠉⠬⠒⠆⠈⠾ $[F = 1.8C + 32]$

(the formula for converting from Centigrade to Fahrenheit temperature is enclosed within brackets)

(4) ⠠⠋ ⠿⠍⠥⠇⠁⠎ ⠷⠂⠾ ⠯ ⠷⠆⠾ ⠮ ⠗⠑⠎⠥⠇⠞
⠊⠎ ⠕⠃⠞⠁⠔⠫⠲

From formulas (1) and (2) the result is obtained.

(Formula numbers enclosed within mathematical parentheses)

(5) ⠈⠾⠰⠴⠘⠂ $]_0^1$

(closing bracket with 0 as subscript and 1 as superscript; this situation arises commonly in evaluating a definite integral, see §70)

(6) ⠷⠂⠾⠠ ⠷⠆⠾⠠ ⠷⠒⠾⠸⠲ (1), (2), (3).

(1, 2, 3, enclosed within parentheses, followed by a period).

b. To enclose words or phrases in the role of a mathematical expression. Such words or phrases are used as if they were mathematical signs — they may be joined with operation signs, they may carry superscripts or subscripts, they may themselves be a superscript or a subscript, or they may occupy the position of the numerator or denominator of a fraction. The contractions which may be used in such words or phrases are discussed in §108f.

(1) ⠷⠗⠁⠞⠑⠾ ⠈⠡ ⠷⠞⠊⠍⠑⠾ ⠨⠅ ⠷⠙⠊⠎⠞⠁⠝⠉⠑⠾

(rate) × (time) = (distance)

(each word is enclosed within parentheses)

(2) ⠷⠞⠓⠑ ⠲⠞⠨⠑ ⠆⠞ ⠰⠠⠁ ⠯ ⠠⠃⠾
⠨⠅ ⠷⠞⠓⠑ ⠲⠞⠨⠑ ⠆⠞ ⠰⠠⠃ ⠯ ⠠⠁⠾

(the distance between A and B) = (the distance between B and A)

(3) ⠍⠐⠕⠽ ⠷⠉⠕⠎⠞ ⠷ ⠕⠝⠑⠾
⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
⠝⠥⠍⠃⠻ ⠃⠕⠥⠛⠓⠞⠕⠍⠕⠝⠑⠽ ⠷⠉⠕⠎⠞ ⠷ ⠁⠇⠇⠾

$$\begin{array}{rl} & \text{money (cost of one)} \\ \text{number bought} & \overline{\big)\ \text{money (cost of all)}} \end{array}$$

§149. Vertical Bars:

a. Double boldface vertical bars are usually read as *the norm of.*

(1) ⠸⠳⠸⠳⠋⠸⠳⠸⠳ $\|f\|$

(the norm of f)

b. Single vertical bars are often read as *the absolute value,* but are used for other purposes.

(1) ⠳⠭⠳ $|x|$

(x enclosed within vertical bars)

(2) ⠳⠰⠴⠘⠂ $\Big|_0^1$

(vertical bar with 0 as subscript and 1 as superscript; this situation occurs commonly in the evaluation of a definite integral)

(3) ⠳⠰⠭ ⠰⠨⠅ ⠰⠁ $\Big|_{x=a}$

(x equals a is subscript to the vertical bar)

c. A single boldface vertical bar is sometimes used to signify *end of proof.*

RULE XIX—OTHER SIGNS

Angstrom unit	Å	⠈⠠⠁	(dots 4, 6, 1)
At	@	⠈⠁	(dots 4, 1)
Caret (circumflex)	^	⠸⠣	(dots 4-5-6, 1-2-6)
Cent	¢	⠈⠉	(dots 4, 1-4)
Crossed d	đ	⠈⠫	(dots 4, 1-2-4-6)
Crossed h	ħ	⠈⠓	(dots 4, 1-2-5)
Decimal	.	⠨	(dots 4-6)
Del (nabla)	▽	⠨⠫	(dots 4-6, 1-2-4-6)
Dollar	$	⠈⠎	(dots 4, 2-3-4)
Empty Set	∅ or ⦶	⠸⠴	(dots 4-5-6, 3-5-6)
Existential Quantifier (there exists, for some)	∃	⠈⠿	(dots 4, 1-2-3-4-5-6)
Factorial	!	⠯	(dots 1-2-3-4-6)
Infinity	∞	⠠⠿	(dots 6, 1-2-3-4-5-6)
Integral			
Single integral	$\int$	⠮	(dots 2-3-4-6)
Double integral	$\int\int$	⠮⠮	(dots 2-3-4-6, 2-3-4-6)
Triple integral	$\int\int\int$	⠮⠮⠮	(dots 2-3-4-6, 2-3-4-6, 2-3-4-6)
Lower integral	$\underline{\int}$	⠩⠮	(dots 1-4-6, 2-3-4-6)
Upper integral	$\overline{\int}$	⠣⠮	(dots 1-2-6, 2-3-4-6)
Integral with superposed circle	$\oint$	⠮⠈⠫⠉	(dots 2-3-4-6, 4, 1-2-4-6, 1-4)
Integral with superposed infinity		⠮⠈⠠⠿	(dots 2-3-4-6, 4, 6, 1-2-3-4-5-6)
Integral with superposed rectangle		⠮⠈⠫⠗	(dots 2-3-4-6, 4, 1-2-4-6, 1-2-3-5)

Integral with superposed square	⨖	⠮⠈⠫⠲	(dots 2-3-4-6, 4, 1-2-4-6, 2-5-6)
Percent	%	⠈⠴	(dots 4, 3-5-6)
Pound Sterling	£	⠈⠇	(dots 4, 1-2-3)
Prescription	℞	⠫⠗⠭	(dots 1-2-4-6, 1-2-3-5, 1-3-4-6)
Prime	′	⠄	(dot 3)
Round d (partial derivative)	∂	⠈⠙	(dots 4, 1-4-5)
Since (because)	∵	⠈⠌	(dots 4, 3-4)
Therefore	∴	⠠⠡	(dots 6, 1-6)
Universal Quantifier (for all, for each, for every)	∀ or ∀	⠈⠯	(dots 4, 1-2-3-4-6)
Vertical Bar (such that)	|	⠳	(dots 1-2-5-6)
Weierstrass p	℘	⠈⠏	(dots 4, 1-2-3-4)

§150. At:

(1) ⠼⠒ ⠃⠕⠭⠑⠎ ⠈⠁ ⠼⠆⠶⠈⠉ 3 boxes @ 27¢

(3 boxes at 27 cents)

§151. Caret:

(1) ⠼⠨⠒⠢⠸⠣⠶⠒ .35 ‸ 73

(decimal point 35, caret 73; this device is sometimes used to show the new location of the decimal point)

§152. Cent, Dollar, Percent, Pound Sterling: In transcribing the *cent* sign, *dollar* sign, *percent* sign or *pound sterling* sign, the corresponding symbols must occupy the same position relative to the numerals with which they are associated as is the case in print; if these signs occupy a position to the right of the associated numeral in print, the corresponding symbol must occupy a position to the right of the associated numeral in braille, and a similar remark applies if they occupy a position to the left of the associated numeral in print.

(1) ⠼⠂⠴⠈⠉ 10¢

(10 cents)

(2) ⠭⠈⠉ x¢

(x cents)

(3) ⠈⠎⠆⠨⠔⠦ $2.98

(2 dollars and 98 cents)

(4) ⠈⠎⠭ $x

(x dollars)

(5) ⠼⠶⠈⠴ 7%

(7 percent)

(6) ⠭⠈⠴ x%

(x percent)

(7) ⠈⠇⠢ £5

(5 pounds)

(8) ⠈⠇⠭ £x

(x pounds)

§153. **Crossed h:** This sign usually denotes Planck's constant divided by 2 pi.

§154. **Decimal:** (See §3 for the use of the numeric indicator; see §96**d** for the use of the multipurpose indicator.)

§155. **Empty Set, (null set, void set):** The transcriber must not mistake this sign for the Greek lower case phi to which it is similar. The publisher of the printed text usually makes the distinction sufficiently clear by the use of easily distinguishable type for the two signs.

§156. **Existential Quantifier:** Sometimes this sign is followed by a vertical bar; the combination then usually means *there exists uniquely.*

(1) ⠈⠿⠳ $\exists|$

(there exists uniquely)

§157. **Factorial:**

(1) ⠝⠯ $n!$

(n factorial)

(2) ⠷⠝⠤⠅⠾⠯ $(n — k)!$

(n minus k factorial)

§158. Infinity:

(1) ⠐⠇⠊⠍⠩⠭⠀⠫⠒⠒⠕⠀⠠⠿⠻⠀⠋⠷⠭⠾ $\lim\limits_{x \longrightarrow \infty} f(x)$

(the limit as x approaches infinity of f of x; arrow and f(x) are spaced from their neighboring signs in print)

(2) ⠮⠰⠴⠘⠠⠿⠐⠋⠷⠭⠾⠀⠙⠭ $\int_0^\infty f(x)\ dx$

(the integral from 0 to infinity of f of x, dx)

§159. Integral: The bar over the integral sign, or the bar under the integral sign, must be transcribed as shown in the above list; the technique for the representation of modified expressions must not be used in these cases; other modifiers, however, must be transcribed in accordance with the technique for the representation of modified expressions (see §77-84).

(1) ⠮⠰⠁⠘⠃⠐⠋⠷⠭⠾⠀⠙⠭ $\int_a^b f(x)\ dx$

(the integral from a to b of f of x, dx)

(2) ⠐⠮⠩⠴⠣⠠⠿⠻⠋⠷⠭⠾⠀⠙⠭ $\int\limits_0^\infty f(x)\ dx$

(the integral from 0 to infinity of f of x, dx)

(3) ⠣⠮⠰⠁⠘⠃⠐⠋⠷⠭⠾⠀⠙⠭⠀⠨⠅⠀⠼⠴ $\overline{\int}_a^b f(x)\ dx = 0$

(the upper integral between a and b of f of x, dx equals 0)

(4) ⠩⠮⠰⠁⠘⠃⠐⠋⠷⠭⠾⠀⠙⠭⠀⠨⠅⠀⠼⠴ $\underline{\int}_a^b f(x)\ dx = 0$

(the lower integral between a and b of f of x, dx equals 0)

(5) ⠐⠮⠩⠭⠨⠅⠁⠣⠭⠨⠅⠃⠻⠋⠷⠞⠾⠀⠙⠞ $\int\limits_{x=a}^{x=b} f(t)\ dt$

(the integral from x equals a to x equals b of f of t, dt)

(6) ⠐⠮⠮⠩⠠⠗⠻⠋⠷⠭⠠⠽⠾⠀⠙⠠⠁ $\iint\limits_R f(x, y)\ dA$

(the double integral over R of f of x and y, dA)

(7) ⠐⠮⠮⠮⠩⠠⠗⠻⠋⠷⠭⠠⠀⠽⠠⠀⠵⠾⠀⠙⠠⠧ $\iiint\limits_R f(x, y, z)\ dV$

(the triple integral over R of f of x, y, and z, dV)

§160. Prime: The single and double primes are often used to denote *feet* and *inches*, respectively. They are also used to denote *minutes* and *seconds*, respectively, whether of time or of angle.

(1) ⠭⠄ x'

(x prime)

(2) ⠭⠄⠄ x''

(x double prime)

(3) ⠭⠄⠄⠄ x'''

(x triple prime)

(4) ⠭⠄⠘⠆ x'^2

(x prime squared)

(5) ⠭⠄⠰⠊ x'_i

(x prime sub i)

(6) ⠭⠄⠂ x'_1

(x prime sub 1)

(7) ⠼⠢⠄⠦⠄⠄ $5' \, 8''$

(5 feet, 8 inches)

(8) ⠼⠆⠴⠘⠨⠡⠐⠒⠴⠄⠂⠴⠄⠄ $20^\circ \; 30' \; 10''$

(20 degrees, 30 minutes, 10 seconds)

§161. Round d, partial derivative:

(1) ⠹⠈⠙⠌⠈⠙⠭⠼⠋⠷⠭⠠ ⠽⠾ $\frac{\partial}{\partial x} f(x, y)$

(the partial with respect to x of f(x, y))

(2) ⠹⠈⠙⠘⠆⠐⠥⠌⠈⠙⠭⠈⠙⠽⠼ $\frac{\partial^2 u}{\partial x \, \partial y}$

(the second partial derivative of u with respect to x and y)

§162. Since, Therefore: The negation of the sign for *therefore* is read *it does not follow that.*

(1) ⠈⠌ ⠠⠁⠠⠃ ⠨⠅ ⠠⠁⠠⠉ $\because AB = AC$

(since AB equals AC)

(2) ⠌⠠⠡ $/\!\therefore$

(it does not follow that)

§163. Universal quantifier:

(1) ⠈⠯⠭ ⠈⠑ ⠠⠁ $\forall x \; \varepsilon \; A$

(for all x belonging to A)

RULE XX—ABBREVIATIONS

Abbreviation	Meaning		
amp	amplitude	⠁⠍⠏	(dots 1, 1-3-4, 1-2-3-4)
antilog	antilogarithm	⠁⠝⠞⠊⠇⠕⠛	(dots 1, 1-3-4-5, 2-3-4-5, 2-4, 1-2-3, 1-3-5, 1-2-4-5)
arc	arc	⠁⠗⠉	(dots 1, 1-2-3-5, 1-4)
arg	argument	⠁⠗⠛	(dots 1, 1-2-3-5, 1-2-4-5)
colog	cologarithm	⠉⠕⠇⠕⠛	(dots 1-4, 1-3-5, 1-2-3, 1-3-5, 1-2-4-5)
cos	cosine	⠉⠕⠎	(dots 1-4, 1-3-5, 2-3-4)
cosh	hyperbolic cosine	⠉⠕⠎⠓	(dots 1-4, 1-3-5, 2-3-4, 1-2-5)
cot	cotangent	⠉⠕⠞	(dots 1-4, 1-3-5, 2-3-4-5)
coth	hyperbolic cotangent	⠉⠕⠞⠓	(dots 1-4, 1-3-5, 2-3-4-5, 1-2-5)
covers	coversine	⠉⠕⠧⠑⠗⠎	(dots 1-4, 1-3-5, 1-2-3-6, 1-5, 1-2-3-5, 2-3-4)
csc	cosecant	⠉⠎⠉	(dots 1-4, 2-3-4, 1-4)
csch	hyperbolic cosecant	⠉⠎⠉⠓	(dots 1-4, 2-3-4, 1-4, 1-2-5)
ctn	cotangent	⠉⠞⠝	(dots 1-4, 2-3-4-5, 1-3-4-5)
ctnh	hyperbolic cotangent	⠉⠞⠝⠓	(dots 1-4, 2-3-4-5, 1-3-4-5, 1-2-5)
erf	error function	⠑⠗⠋	(dots 1-5, 1-2-3-5, 1-2-4)
exp	exponential	⠑⠭⠏	(dots 1-5, 1-3-4-6, 1-2-3-4)
exsec	exsecant	⠑⠭⠎⠑⠉	(dots 1-5, 1-3-4-6, 2-3-4, 1-5, 1-4)
hav	haversine	⠓⠁⠧	(dots 1-2-5, 1, 1-2-3-6)
inf	infimum	⠊⠝⠋	(dots 2-4, 1-3-4-5, 1-2-4)

Abbreviation	Meaning		
lim	limit	⠇⠊⠍	(dots 1-2-3, 2-4, 1-3-4)
$\overline{\text{lim}}$	upper limit	⠣⠇⠊⠍	(dots 1-2-6, 1-2-3, 2-4, 1-3-4)
$\underline{\text{lim}}$	lower limit	⠩⠇⠊⠍	(dots 1-4-6, 1-2-3, 2-4, 1-3-4)
ln	natural logarithm	⠇⠝	(dots 1-2-3, 1-3-4-5)
log	logarithm	⠇⠕⠛	(dots 1-2-3, 1-3-5, 1-2-4-5)
max	maximum	⠍⠁⠭	(dots 1-3-4, 1, 1-3-4-6)
min	minimum	⠍⠊⠝	(dots 1-3-4, 2-4, 1-3-4-5)
mod	modulo	⠍⠕⠙	(dots 1-3-4, 1-3-5, 1-4-5)
sec	secant	⠎⠑⠉	(dots 2-3-4, 1-5, 1-4)
sech	hyperbolic secant	⠎⠑⠉⠓	(dots 2-3-4, 1-5, 1-4, 1-2-5)
sin	sine	⠎⠊⠝	(dots 2-3-4, 2-4, 1-3-4-5)
sinh	hyperbolic sine	⠎⠊⠝⠓	(dots 2-3-4, 2-4, 1-3-4-5, 1-2-5)
sup	supremum	⠎⠥⠏	(dots 2-3-4, 1-3-6, 1-2-3-4)
tan	tangent	⠞⠁⠝	(dots 2-3-4-5, 1, 1-3-4-5)
tanh	hyperbolic tangent	⠞⠁⠝⠓	(dots 2-3-4-5, 1, 1-3-4-5, 1-2-5)
vers	versine	⠧⠑⠗⠎	(dots 1-2-3-6, 1-5, 1-2-3-5, 2-3-4)

§164. Abbreviations of Measurement:

a. Abbreviations of measurement must be transcribed as they appear in print, except that periods must be omitted unless the final period ends a sentence. The distinction between lower-case and capitalized letters must be preserved in the transcription. Contractions must not be used in abbreviations of measurement.

(1) ⠼⠒ ⠋⠞ ⠨⠅ ⠼⠂ ⠽⠙⠸⠲ 3 ft. = 1 yd.

(3 feet equals 1 yard)

(2) ⠼⠆⠢ ⠎⠟ ⠋⠞ 25 sq. ft.

(25 square feet; the s and f are lower case in print)

(3) ⠼⠆⠢ ⠠⠎⠟ ⠠⠋⠞ 25 Sq. Ft.

(25 square feet; the s and f are capitalized in print)

(4) ⠼⠂⠢ ⠊⠝ 15 in

(15 inches)

(5) ⠼⠆⠒ ⠍⠊⠝ 23 min

(23 minutes)

b. If a measurement is not abbreviated, but is expressed as a whole word, the contractions of English Braille may be used. However, if a whole-word measurement is part of a compound abbreviation in which the diagonal line or the hyphen occur, or if it carries a superscript or a subscript, no contractions may be used.

(1) ⠼⠔⠦⠴ ⠻⠛⠎ 980 ergs

(2) ⠼⠂ ⠺⠁⠞⠞ ⠨⠅ ⠼⠂ ⠚⠕⠥⠇⠑⠸⠌⠎⠑⠉

1 watt = 1 joule/sec

(1 watt equals 1 joule per second)

c. When *feet* and *minutes* are represented by the single prime, or when *inches* and *seconds* are represented by the double prime, the primes, and not the abbreviations, must be used (see §160).

§165. Literary Abbreviations: In literary abbreviations, as opposed to abbreviations of measurement, the period must not be omitted. It should, however, be preceded by the punctuation indicator if the abbreviation before the period consists of a single letter.

(1) ⠁⠸⠲⠍⠸⠲ a.m.

(before noon)

(2) ⠏⠸⠲⠍⠸⠲ p.m.

(after noon)

(3) ⠑⠸⠲⠛⠸⠲ e.g.

(for example)

(4) ⠊⠸⠲⠑⠸⠲ i.e.

(that is)

(5) ⠠⠞⠥⠗⠝ ⠞⠕ ⠰⠏⠸⠲ ⠼⠃⠛⠸⠲ Turn to p. 27.

(Turn to page 27)

(6) ⠠⠒⠎⠥⠇⠞ ⠏⠏⠲ ⠼⠉⠓⠤⠙⠑ Consult pp. 38-45

(Consult pages 38-45)

(7) ⠠⠎⠑⠑ ⠠⠋⠊⠛⠲ ⠼⠂⠴ See Fig. 10
(See figure 10)

(8) ⠠⠇⠸⠲⠠⠥⠸⠲⠠⠃⠸⠲ L.U.B.
(least upper bound)

(9) ⠠⠛⠸⠲⠠⠇⠸⠲⠠⠃⠸⠲ G.L.B.
(greatest lower bound)

(10) ⠼⠂⠂⠴ ⠠⠺⠸⠲ ⠼⠂⠂⠴⠞⠓ ⠠⠌⠲ 110 W. 110th St.
(110 West 110th Street)

§166. **Abbreviated Scientific Words and Phrases:** An abbreviated scientific word which does not appear in the above list must be transcribed in accordance with the rules which follow:

a. Contractable letter combinations which occur in an abbreviated scientific word must not be contracted.

b. The capitalization of the initial letter in an abbreviated scientific word must be indicated in the transcription if the corresponding letter is capitalized in print.

(1) ⠠⠁⠗⠉ ⠎⠊⠝ ⠭ Arc sin x
(the inverse sine of x)

§167. **Logarithms:** When the base of a logarithm is explicitly printed, it must be treated in braille, as in print, as a subscript.

(1) ⠇⠕⠛⠰⠁ ⠭ $\log_a x$
(logarithm of x to the base a)

(2) ⠇⠕⠛⠆ ⠭ $\log_2 x$
(logarithm of x to the base 2)

§168. **Trigonometric and Hyperbolic Functions:** The list shows two alternate abbreviations in common use for the *cotangent* and the *hyperbolic cotangent*. The same abbreviated forms must be used in braille as appear in print.

§169. **Limit:** The bar which occurs over or under the abbreviated or fully spelled out word *limit* must not be treated as a modifier; the combination must be transcribed by means of the special symbols for *upper limit* or *lower limit* in the list. Other modifiers, however, must be transcribed in accordance with the technique for the representation of modified expressions (see §77-84).

(1) ⠐⠇⠊⠍⠩⠭ ⠫⠒⠒⠕ ⠼⠴⠻ ⠋⠷⠭⠾ $\lim\limits_{x \to 0} f(x)$
(the limit, as x approaches 0, of f of x)

(2) ⠐⠣⠇⠊⠍⠩⠝ ⠫⠒⠒⠕ ⠠⠿⠻ ⠋⠰⠝⠐⠷⠭⠾ $\overline{\lim}_{n\to\infty}\ f_n\ (x)$

(the upper limit, as n approaches infinity, of f sub n of x)

(3) ⠐⠩⠇⠊⠍⠩⠝ ⠫⠒⠒⠕ ⠠⠿⠻ ⠋⠰⠝⠐⠷⠭⠾ $\underline{\lim}_{n\to\infty}\ f_n\ (x)$

(the lower limit, as n approaches infinity, of f sub n of x)

RULE XXI—SIGNS AND SYMBOLS OF FORMAT

The numeric indicator must not be used with numerals associated with any of the symbols in the following list:

Division Format			
Inverted with curved side	$\underline{)}$	⠕ ⠒⠒⠒	(dots 1-3-5 on one line, separation line below)
Inverted with straight side	$\underline{\vert}$	⠳ ⠒⠒⠒	(dots 1-2-5-6 on one line, separation line below)
Normal with curved side	$\overline{)}$	⠒⠒⠒ ⠕	(dots 1-3-5 on one line, separation line above)
Normal with straight side	$\overline{\vert}$	⠒⠒⠒ ⠳	(dots 1-2-5-6 on one line, separation line above)
Separation Line	—		
Ordinary		⠒⠒⠒	(dots 2-5, 2-5, 2-5)
For changing number form (carrying or borrowing)		⠶⠶⠶	(dots 2-3-5-6, 2-3-5-6, 2-3-5-6)
Square Root Format	$\sqrt{}$	⠒⠒⠒ ⠜	(dots 3-4-5 on one line, separation line above)

Synthetic Division Format

Normal

⠠⠳
⠠⠳
⠒⠒⠒⠒

(dots 6, 1-2-5-6 at left of two lines, separation line below)

Reverse

⠠⠳
⠠⠳
⠒⠒⠒⠒

(dots 6, 1-2-5-6 at right of two lines, separation line below)

§170. Division Format:

a. The separation line which is part of a division format symbol must not be omitted, even when no quotient appears in the printed text. This situation is common in arithmetic textbooks where the student is presented with a list of division problems; the division forms are indicated, but no quotients are supplied. This separation line must begin at the division symbol and extend one cell to the right of material immediately above or below it (see also §171c.)

(1)
⠒⠒⠒⠒⠒
⠆⠢⠕⠲⠢⠆

25)452

(25 divided into 452)

b. When there is a remainder associated with a division problem, it is a common practice to place the remainder on the same line with the quotient and to use the letter r before the numeral which specifies the remainder. This practice must also be followed in the transcription when it is present in print. The r must be preceded by a space. The r must be lower case or capitalized in accordance with its printed form. The multipurpose indicator must follow the r and precede the numeral which specifies the remainder to indicate that this numeral is not a subscript to r.

(1)
⠂⠦ ⠗⠐⠆
⠒⠒⠒⠒⠒⠒⠒⠒⠒
⠆⠢⠕⠲⠢⠆

18 r2
25)452

(25 divided into 452 yields a quotient of 18 and a remainder of 2; normal division format)

(2)
⠆⠢⠕⠲⠢⠆
⠒⠒⠒⠒⠒⠒⠒⠒⠒
⠂⠦ ⠗⠐⠆

25)452
18 r2

(as in (1) above using the inverted division format with curved side)

(3)

```
        ⠂⠦ ⠗⠐⠆
    ⠒⠒⠒⠒⠒⠒⠒⠒⠒
⠆⠢⠳⠲⠢⠆
```

$$\begin{array}{r|l} & 18\text{ r}2 \\ \cline{2-2} 25 & 452 \end{array}$$

(as in (1) above using the normal division format with straight side)

(4)

```
⠆⠢⠳⠲⠢⠆
    ⠒⠒⠒⠒⠒⠒⠒⠒⠒
        ⠂⠦ ⠗⠐⠆
```

$$\begin{array}{r|l} 25 & 452 \\ \cline{2-2} & 18\text{ r}2 \end{array}$$

(as in (2) above, using the inverted division format with straight side)

(5)

```
  ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
⠖⠕⠲⠂⠖ ⠸ ⠂⠴
    ⠖⠴ ⠸
  ⠒⠒⠒⠒ ⠸
  ⠒⠢⠖ ⠸ ⠆⠴
  ⠂⠆⠴ ⠸
 ⠒⠒⠒⠒⠒⠸
  ⠆⠒⠖ ⠸ ⠒⠴
  ⠂⠦⠴ ⠸
 ⠒⠒⠒⠒⠒⠸
   ⠢⠖ ⠸  ⠔
   ⠢⠲ ⠸
  ⠒⠒⠒⠒⠸
    ⠆ ⠸ ⠖⠔⠸⠹⠂⠌⠒⠸⠼
```

$$\begin{array}{r|l} 6\overline{)416} & 10 \\ 60 & \\ \hline 356 & 20 \\ 120 & \\ \hline 236 & 30 \\ 180 & \\ \hline 56 & 9 \\ 54 & \\ \hline 2 & 69\frac{1}{3} \end{array}$$

(the vertical line in this example may be represented by a column of dots 4-5-6, or it may be drawn in)

§171. Separation Line:

a. In work arranged for addition, subtraction, or multiplication, all separation lines in braille must extend in both directions one cell beyond the over-all width of the arrangement (see §183).

(1)

```
⠀⠀⠆⠢
⠀⠬⠀⠶
⠒⠒⠒⠒⠒
⠀⠀⠒⠆
```

$$\begin{array}{r} 25 \\ +\ 7 \\ \hline 32 \end{array}$$

(25 plus 7 equals 32)

(2)

```
⠀⠀⠂⠴⠠⠴⠴⠴
⠀⠤⠀⠔⠠⠴⠴⠴
⠒⠒⠒⠒⠒⠒⠒⠒⠒
⠀⠀⠀⠂⠠⠴⠴⠴
```

$$\begin{array}{r} 10{,}000 \\ -\ 9{,}000 \\ \hline 1{,}000 \end{array}$$

(10,000 minus 9,000 equals 1,000)

(3)

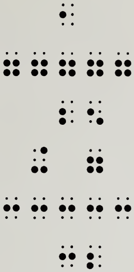
```
⠀⠀⠀⠆⠒
⠀⠈⠡⠢⠲
⠒⠒⠒⠒⠒⠒
⠀⠀⠀⠔⠆
⠀⠂⠂⠢
⠒⠒⠒⠒⠒⠒
⠀⠂⠆⠲⠆
```

$$\begin{array}{r} 23 \\ \times\ 54 \\ \hline 92 \\ 115 \\ \hline 1242 \end{array}$$

(23 times 54 equals 1242)

b. In addition or subtraction problems in which carries are indicated, a separation line, even if not present in print, must be used in the transcription. The separation line used for this purpose consists of a series of dots 2-3-5-6. It must be placed above the first line of the problem and must be as long as the ordinary separation line. The carried or borrowed numbers are placed above this separation line in the appropriate columns.

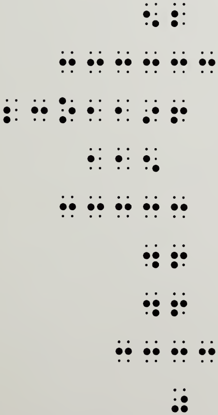
(1)



$$\begin{array}{r} {}^{1} \\ 25 \\ +\ 7 \\ \hline 32 \end{array}$$

c. In work arranged for division or the extraction of a root, the separation lines, except the format separation line (see §170a), in braille must extend in both directions one cell beyond the line immediately *above* it, unless a vertical barrier prevents its extension in one direction.

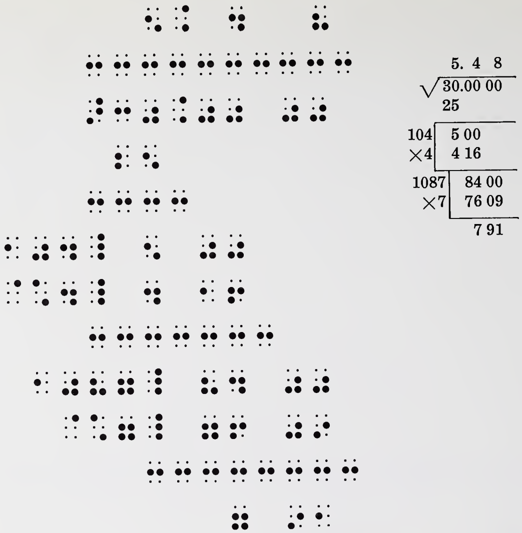
(1)



$$\begin{array}{r} 52 \\ 23\overline{\smash{)}1196} \\ 115 \\ \hline 46 \\ 46 \\ \hline 0 \end{array}$$

(23 divided into 1196 yields a quotient of 52; a long division problem)

(2)

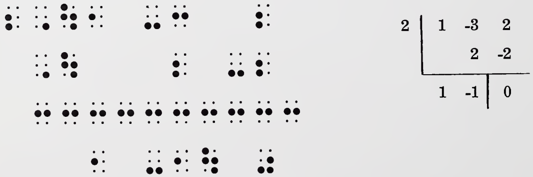


(the square root of 30, rounded to two decimal places, equals 5.48 (see also §170b(5)).

§172. **Square Root Format:** (See §171c(2).)

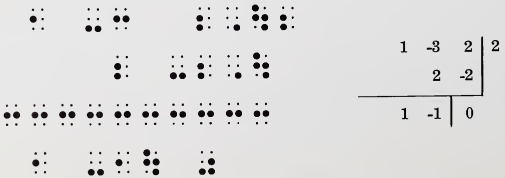
§173. **Synthetic Division Format:** (See §170b(5).)

(1)



(x — 2 divided into x² — 3x + 2 yields a quotient of x — 1 and a remainder of 0)

(2)



(as in (1) above, using the reverse synthetic division format)

§174. **Special Arrangement for Multiplication:** If commas or the decimal point are to be omitted in braille from the partial products of a product arranged for computation, the decimal point or the commas should be placed below the product. In this case, the decimal point should be placed under the units digit of the product, and the commas should be placed under the thousands, millions, etc., digits of the product. If provision is to be made for commas or the decimal point in braille in the partial products, the spacing between the digits of the partial products, as well as in the final product, must be such as to permit their insertion.

(1)

```
  ⠒⠲⠢⠨⠶
 ⠈⠡⠆⠨⠶⠶
⠒⠒⠒⠒⠒⠒⠒⠒
  ⠆⠲⠂⠔⠔
 ⠆⠲⠂⠔⠔
 ⠖⠔⠂⠲
⠒⠒⠒⠒⠒⠒⠒⠒
 ⠔⠢⠶⠢⠦⠔
   ⠨
```

```
    345.7
    ×2.77
   ------
    24199
   24199
   6914
   ------
   957589
     .
```

(2)

```
   ⠒⠲⠢⠨⠶
  ⠈⠡⠆⠨⠶⠶
⠒⠒⠒⠒⠒⠒⠒⠒⠒
  ⠆⠲ ⠂⠔⠔
 ⠆⠲⠂ ⠔⠔
 ⠖⠔⠂ ⠲
⠒⠒⠒⠒⠒⠒⠒⠒⠒
 ⠔⠢⠶⠨⠢⠦⠔
```

```
     345.7
     ×2.77
   -------
    24 199
   241 99
   691 4
   -------
   957.589
```

RULE XXII—SPACING

§175. **Spacing with Symbols of Operation:** Except as indicated in **a-d** below, no space should be left either before or after an operation symbol.

a. A space must be left before the plus symbol or minus symbol if it terminates an argument associated with an abbreviated scientific word or phrase.

(1) ⠎⠊⠝⠀⠭⠀⠬⠎⠊⠝⠀⠽ sin x + sin y

(sine x plus sine y)

b. A space must be left both before and after an operation symbol if it is also a shape symbol, modified or unmodified.

(1) ⠭⠀⠫⠉⠨⠬⠀⠽ x ⊕ y

(circle with interior plus sign between x and y)

(2) ⠭⠀⠫⠲⠀⠽ x □ y

(hollow square between x and y)

(3) ⠭⠀⠫⠸⠲⠀⠽ x ■ y

(filled-in square between x and y)

(4) ⠫⠲⠀⠬⠀⠫⠒ □ + △

(square plus triangle)

c. A space must be left between a symbol of operation and a word in the role of a mathematical expression.

(1) ⠗⠁⠞⠑⠀⠈⠡⠀⠐⠞ rate × time

(cross between *rate* and *time*)

(2) ⠼⠨⠶⠈⠡⠀⠍⠁⠭⠊⠍⠥⠍⠀⠉⠥⠗⠗⠢⠞

.7 × maximum current

d. One or more spaces may be left on either side of an operation symbol when a body of mathematical expressions is arranged so that operation signs fall into the same vertical column. If a body of mathematical expressions is arranged as just described, or if the comparison signs also fall into a vertical column, this arrangement must be preserved in the transcription. If it is necessary to leave one or more spaces between the corresponding symbols of operation or of comparison and the other mathematical expressions to achieve such an arrangement in braille, this is of no consequence.

(1)

⠀⠀⠆⠭⠘⠒⠐⠤⠀⠭⠘⠆⠐⠬⠀⠀⠭⠬⠀⠂
⠀⠀⠒⠭⠘⠒⠐⠬⠲⠭⠘⠆⠐⠤⠂⠴⠭⠬⠀⠶
⠀⠀⠀⠀⠀⠀⠀⠀⠢⠭⠘⠆⠐⠬⠀⠀⠀⠀⠂⠆
⠀⠤⠆⠭⠘⠒⠐⠤⠀⠀⠀⠀⠀⠀⠀⠖⠭
⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
⠀⠀⠒⠭⠘⠒⠐⠬⠦⠭⠘⠆⠐⠤⠂⠢⠭⠬⠆⠴

$$\begin{array}{rrrr} 2x^3 - & x^2 + & x + & 1 \\ 3x^3 + & 4x^2 - & 10x + & 7 \\ & 5x^2 + & & 12 \\ -2x^3 - & & 6x & \\ \hline 3x^3 + & 8x^2 - & 15x + & 20 \end{array}$$

(an addition problem in algebra arranged for actual computation)

(2)
⠥ ⠨⠅ ⠭⠬⠽⠬⠵⠠
⠭⠬⠥ ⠨⠅ ⠁⠬⠃⠠
⠽⠬⠥ ⠨⠅ ⠁⠬⠉⠠
⠵⠬⠥ ⠨⠅ ⠁⠬⠙⠸⠲

$$\begin{aligned} u &= x + y + z, \\ x + u &= a + b, \\ y + u &= a + c, \\ z + u &= a + d. \end{aligned}$$

(a system of four equations)

§176. Spacing with Symbols of Comparison:

a. Except for punctuation (see §178c), a space must generally be left on both sides of a comparison symbol unless it is *clear* from the printed text that no space should be left. When there is doubt concerning the presence of a space in print, a space should be left in the transcription. Transition to another braille line takes the place of this space. In any case, symbols of comparison *which are also symbols of shape* require a space on both sides. Unspaced comparison signs occur most frequently when they are part of a modifier directly under a mathematical expression.

(1) ⠐⠇⠊⠍⠩⠭ ⠫⠒⠒⠕ ⠼⠴⠻ ⠋⠷⠭⠾

$$\lim_{x \to 0} f(x)$$

(the limit, as x approaches 0, of f of x; space on both sides of arrow in braille even though the arrow in print is unspaced from its neighboring signs)

(2) ⠐⠨⠠⠎⠩⠝ ⠨⠅ ⠼⠴⠣⠠⠿⠻⠁⠰⠝

$$\sum_{n=0}^{\infty} a_n$$

(the summation, from n equals 0 to infinity, of a sub n)

(3) ⠐⠨⠠⠎⠩⠊⠐⠅⠚⠻⠁⠰⠊⠚

$$\sum_{i<j} a_{ij}$$

(the summation, with i less than j, of a sub ij)

b. When a comparison symbol is associated with a system of mathematical expressions which is unified by an enlarged grouping symbol, the comparison symbol should be placed midway between the top and bottom of the unified mathematical expression. If the transcription contains an even number of lines for such a unified mathematical expression, the comparison symbol must be placed on the upper of the two middle lines. In particular, if the unified mathematical expression occupies two lines, the associated comparison symbol must be placed on the upper of these two lines.

(1)
⠠⠙ ⠨⠅ ⠠⠳⠁⠂⠂ ⠁⠂⠆⠠⠳ ⠨⠅ ⠼⠂⠴⠴
⠠⠳⠁⠆⠂ ⠁⠆⠆⠠⠳

$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 100$$

(a two-by-two determinant)

(2)
⠠⠳⠁⠂⠂ ⠁⠂⠆ ⠁⠂⠒⠠⠳
⠠⠙ ⠨⠅ ⠠⠳⠁⠆⠂ ⠁⠆⠆ ⠁⠆⠒⠠⠳
⠠⠳⠁⠒⠂ ⠁⠒⠆ ⠁⠒⠒⠠⠳

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

(a three-by-three determinant)

(3)

⠀⠀⠀⠀⠀⠀⠀⠀⠀⠠⠳⠁⠂⠂ ⠁⠂⠆ ⠁⠂⠒ ⠁⠂⠲⠠⠳
⠠⠙ ⠨⠅ ⠠⠳⠁⠆⠂ ⠁⠆⠆ ⠁⠆⠒ ⠁⠆⠲⠠⠳
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠠⠳⠁⠒⠂ ⠁⠒⠆ ⠁⠒⠒ ⠁⠒⠲⠠⠳
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠠⠳⠁⠲⠂ ⠁⠲⠆ ⠁⠲⠒ ⠁⠲⠲⠠⠳

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}$$

(a four-by-four determinant)

§177. **Spacing with Symbols of Grouping:** A left grouping symbol must be followed without a space and a right grouping symbol must be preceded without a space, by the mathematical expressions with which they are respectively associated. This rule applies equally to words in the role of mathematical expressions in which case contractions must not be used. A right grouping symbol may never be disassociated from the mathematical symbol which precedes it by transition to another braille line.

To comply with the above rule, an enlarged grouping symbol must be regarded as being composed of individual components, one on each line containing the unified mathematical expression. While an individual component may be separated from its associated mathematical expression, the enlarged grouping symbol as a whole may not be so separated.

(1) ⠨⠠⠷⠆⠭⠬ ⠽ ⠨⠅ ⠼ ⠔⠨⠠⠾
⠨⠠⠷ ⠭⠤⠒⠽ ⠨⠅ ⠼⠂⠂⠨⠠⠾

$$\left\{\begin{array}{r} 2x + y = 9 \\ x - 3y = 11 \end{array}\right\}$$

(a unified system of two equations)

§178. **Spacing with Symbols of Shape:** Except as indicated in a-c below, a space must be left on both sides of a symbol of shape.

a. No space must be left before or after a symbol of shape if it is preceded or followed, respectively, by a braille indicator.

(1) ⠭⠘⠫⠲ $x^{\square}$

(x with a square as superscript; the symbol for *square* is preceded by the superscript indicator)

(2) ⠫⠣⠒⠒⠕⠐⠫⠩⠒⠒⠕ $\uparrow\downarrow$

(arrow north followed by arrow south; multipurpose indicator before the second arrow to form a symbol of comparison compounded horizontally)

(3) ⠹⠫⠲ ⠌ ⠫⠉⠼ $\frac{\square}{\bigcirc}$

(simple fraction with a square in the numerator and a circle in the denominator; opening fraction indicator precedes the square, closing fraction indicator follows the circle)

(4) ⠫⠲⠸⠲ $\square$.

(square followed by period; no space after the square because it is followed by the punctuation indicator)

(5) ⠐⠃⠣⠫⠒⠒⠕⠻ $\vec{b}$

(arrow over b; arrow is preceded by the *directly over* indicator)

b. No space must be left before or after a symbol of shape if it is associated with another symbol of shape to form a symbol of comparison compounded vertically.

(1) ⠫⠒⠒⠕⠫⠪⠒⠒ $\rightleftarrows$
(arrow east over arrow west)

c. No space must be left before or after a comparison or shape symbol if it is preceded or followed by punctuation associated with that symbol.

(1) ⠦⠫⠲⠸⠴ “□”
(a square enclosed by outer quotations)

(2) ⠨⠳⠫⠞ †△
(a triangle preceded by a dagger)

(3) ⠬⠠ ⠤⠠ ⠨⠅⠸⠆ +, —, =;
(plus sign followed by a comma, minus sign followed by a comma, equals sign followed by a semicolon)

(4) ⠨⠂⠠ ⠐⠅⠸⠆ >, <;
(greater than sign followed by a comma, less than sign followed by a semicolon)

(5) ⠫⠲⠠ ⠫⠉⠸⠲ □, ○.
(square followed by a comma, circle followed by a period)

§179. **Spacing with Abbreviations of Measurement:** An abbreviation of measurement must be spaced from its associated quantity by a space, unless there is a clear indication from the printed text that it should not be spaced. It is permissible to separate an abbreviation of measurement from its associated quantity by transition to another line.

(1) ⠼⠖⠴ ⠍⠊⠸⠌⠓⠗ 60 mi/hr
(60 miles per hour)

(2) ⠼⠂ ⠛⠍⠤⠉⠍⠸⠌⠎⠑⠉⠘⠆ 1 gm-cm/sec^2
(1 gram-centimeter per second squared)

(3) ⠼⠂ ⠇⠊ ⠨⠅ ⠼⠂⠴⠴⠴⠉⠉ 1 li = 1000cc
(1 liter equals 1000 cubic centimeters)

(4) ⠼⠴⠘⠨⠡⠐⠠⠉ ⠨⠅ ⠼⠒⠆⠘⠨⠡⠐⠠⠋ 0°C = 32°F
(0 degrees Centigrade equals 32 degrees Fahrenheit)

§180. Spacing with Abbreviated Scientific Words:

a. Abbreviated scientific words which occur consecutively must be separated by a space.

(1) ⠇⠕⠛ ⠎⠊⠝ ⠭ log sin x
(the logarithm of the sine of x)

(2) ⠣⠇⠊⠍⠀⠎⠥⠏⠀⠨⠷⠁⠰⠝⠐⠨⠾ $\overline{\text{lim}}\ \text{sup}\ \{a_n\}$

(the upper limit of the sequence with general term a sub n)

b. The spacing on the left of an abbreviated scientific word is the same as with any other mathematical expression.

(1) ⠼⠆⠎⠊⠝⠀⠭ $2\sin x$

(2 sine x)

(2) ⠎⠊⠝⠀⠭⠀⠬⠎⠊⠝⠀⠽ $\sin x + \sin y$

(sine of x plus sine of y)

(3) ⠼⠆⠎⠊⠝⠀⠭⠀⠬⠒⠎⠊⠝⠀⠽ $2\sin x + 3\sin y$

(2 sine x plus 3 sine y)

(4) ⠽⠎⠊⠝⠀⠭⠀⠬⠭⠎⠊⠝⠀⠽ $y\sin x + x\sin y$

(y sine x plus x sine y)

§181. **Argument:** An argument is an expression which follows an abbreviated scientific word, exclusive of any superscripts, subscripts, or modifiers which the abbreviated scientific word may carry. In §169(1), f(x) is the argument; in §169 (2) and (3), f_n (x) is the argument. The numeric indicator must not be used before a numeral with which an argument begins. Since the proper implementation of the spacing rules in §182 depends upon the ability to identify the precise point at which an argument ends, the following criteria are presented:

a. An argument is terminated by a clear space in print, or by a change in level. If a space terminates an argument under the effect of a previous level indicator, the new level indicator must be stated even if it is for the base line, unless the next symbol following the argument is a comparison symbol, on the base line.

(1) ⠎⠊⠝⠀⠭⠀⠙⠭ $\sin x\ dx$

(sine of x, dx; a clear space in print between the x and the dx)

(2) ⠑⠘⠎⠊⠝⠀⠭⠀⠐⠬⠑⠘⠎⠊⠝⠀⠽ $e^{\sin x} + e^{\sin y}$

(e to the sine of x power plus e to the sine of y power; the change of level from the x to the plus sign terminates the argument, x)

(3) ⠑⠘⠎⠊⠝⠀⠭⠀⠨⠅⠀⠽ $e^{\sin x} = y$

(e to the sine of x power equals y. No change of level indicators required)

(4) ⠑⠘⠎⠊⠝⠀⠭⠀⠘⠨⠅⠀⠘⠁⠀⠨⠂⠀⠽ $e^{\sin x = a} > y$

(e to the sine x equals a power is greater than y, the space following the letter a restores the base line)

(5) ⠎⠊⠝⠀⠨⠏⠀⠸⠌⠒ $\sin \pi\ /3$

(sine of π over 3; a clear space in print before the diagonal line; pi is the argument)

b. An argument is terminated by a mark of punctuation, by the resumption of literary text, or by transition to another line of braille.

(1) ⠎⠊⠝⠀⠭⠠⠀⠉⠕⠎⠀⠽ $\sin x, \cos y$

(sine of x, cosine of y)

c. Except as in **d** below, an argument is terminated by a plus sign, minus sign, or comparison sign.

(1) ⠎⠊⠝ ⠭ ⠬⠎⠊⠝ ⠽ $\sin x + \sin y$
(sine of x plus sine of y)

(2) ⠎⠊⠝ ⠭ ⠐⠅ ⠭ $\sin x < x$
(sine of x is less than x)

d. An argument is not terminated by a plus sign or minus sign if these signs are part of an enclosed expression in which the left sign of grouping belongs to the argument.

(1) ⠷⠎⠊⠝ ⠭ ⠬⠎⠊⠝ ⠽⠾⠘⠆ $(\sin x + \sin y)^2$
(sine of x plus sine of y, squared; the plus sign is part of an enclosed expression, but the left sign of grouping does not belong to the argument)

(2) ⠎⠊⠝ ⠷⠭⠬⠽⠾ $\sin (x + y)$
(sine of x plus y; the plus sign is part of an enclosed expression, but the left sign of grouping belongs to the argument)

e. An argument is terminated by another abbreviated scientific word.

(1) ⠎⠊⠝ ⠆⠭ ⠨⠅ ⠼⠆⠎⠊⠝ ⠭ ⠉⠕⠎ ⠭ $\sin 2x = 2 \sin x \cos x$
(sine of 2x equals 2 times the sine of x times the cosine of x)

§182. Spacing Rule for Arguments:

a. A space must be left before an argument associated with an abbreviated scientific word. The beginning of an argument must never be separated from its associated abbreviated scientific word by transition to another line of braille. The space which *precedes* an argument does not terminate the effect of the level indicator (see §73**e**); however the space which *follows* an argument terminates the effect of the level indicator (see §181a (3)-(4)).

(1) ⠑⠘⠎⠊⠝ ⠭ $e^{\sin x}$
(e to the sine of x power; the x is at the superscript level)

b. A space must be left after an argument associated with an abbreviated scientific word. However, the transition to another line of braille takes the place of this required space. This required space or transition to another line of braille must precede the plus sign, minus sign, or comparison sign which terminates the argument, but must follow a mark of punctuation or right sign of grouping which terminates the argument. The space which follows an argument terminates the effect of a level indicator. All of the examples in §180-181 illustrate the spacing rule for abbreviated scientific words. Additional examples are:

(1) ⠎⠊⠝ ⠨⠏⠸⠌⠒ $\sin \pi/3$
(sine of π over 3; no clear space in print between pi and the diagonal line)

(2) ⠎⠊⠝ ⠷⠁⠬⠃⠾⠭ $\sin (a + b)x$
(sine of a plus b times x; x belongs to the argument)

(3) ⠹⠎⠊⠝ ⠭ ⠌⠆⠼ $\dfrac{\sin x}{2}$

(sine x, over 2; a space is required after the argument, x, since the simple fraction line is not part of the argument)

(4) ⠹⠎⠊⠝ ⠭ ⠌⠎⠊⠝ ⠽⠼ $\dfrac{\sin x}{\sin y}$

(comment as in (3) above)

(5) ⠎⠊⠝ ⠹⠭⠬⠽⠌⠭⠤⠽⠼ $\sin \dfrac{x+y}{x-y}$

(sine of the fraction x plus y over x minus y; the entire fraction is the argument)

(6) ⠎⠊⠝⠘⠆ ⠭ $\sin^2\ x$

(sine squared of x)

(7) ⠎⠊⠝⠘⠤⠂ ⠭ $\sin^{-1} x$

(the inverse sine of x)

(8) ⠠⠎⠊⠝⠘⠤⠂ ⠭ $\mathrm{Sin}^{-1} x$

(the principal value of the inverse sine of x; the S is capitalized to signify the principal value)

(9) ⠑⠘⠎⠊⠝ ⠭ ⠘⠬⠎⠊⠝ ⠽ $e^{\sin x + \sin y}$

(e to the sine x plus the sine of y power; the superscript indicator must be restated before the plus sign since the space after the x terminates the effect of the superscript indicator)

§183. **Spacing with Separation Lines:** One space is sufficient between separation lines which belong to different problems but which appear on the same line of braille.

(1)

```
  ⠆⠢    ⠂⠢
 ⠬ ⠶   ⠬⠆⠒
⠒⠒⠒⠒⠒ ⠒⠒⠒⠒⠒
  ⠒⠆    ⠒⠦
```

$$\begin{array}{r} 25 \\ +\ 7 \\ \hline 32 \end{array} \qquad \begin{array}{r} 15 \\ +23 \\ \hline 38 \end{array}$$

(25 plus 7 equals 32, 15 plus 23 equals 38; two adjacent addition problems)

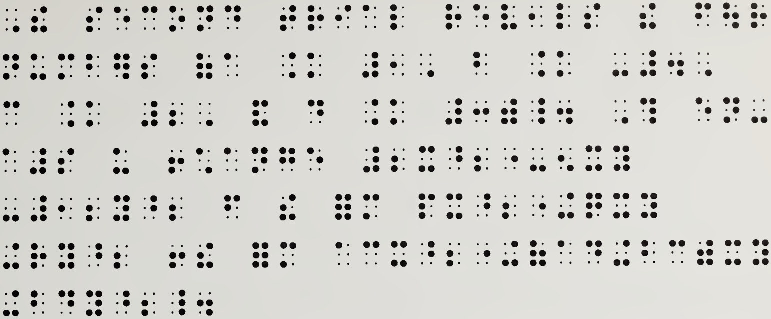
RULE XXIII—RUNOVER AND SPATIAL ARRANGEMENTS

§184. Runovers of Mathematical Expressions:

a. *The runover of a mathematical expression to another braille line should be avoided, if possible.*

b. When a mathematical expression *which is not separated from the text by skipped lines, indentation, or other means* (that is, which is part of the text) must be divided, the runover must begin at the margin. However, the place at which the division is made is determined according to the rules in f below.

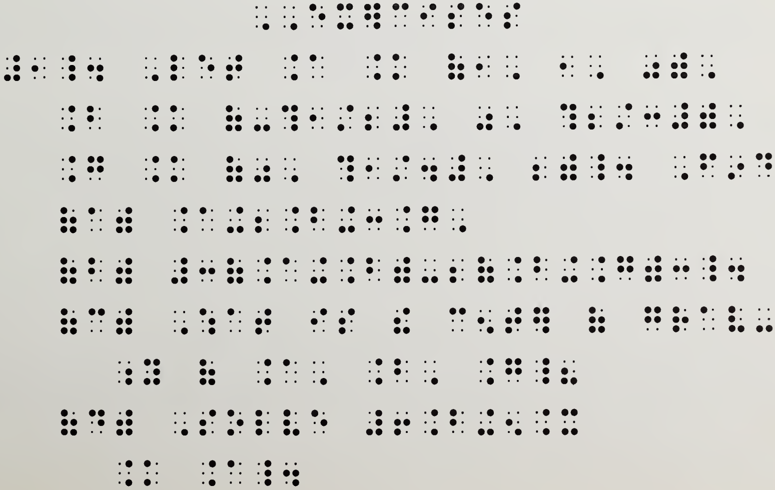
(1)



The second trial reveals the four numbers ($a = 1$, $b = -4$, $c = 2$, and $d = 3$). This enables us to change $2x^2 - 5xy - 12y^2$ from the form $px^2 + qxy + ry^2$ to the form $acx^2 + (ad + bc)xy + bdy^2$.

c. In a set of exercises, the exercise number must begin at the margin and any runover must begin in the third cell. If an exercise is subdivided, the number or letter which identifies the subdivision must generally begin in cell three and run over, if necessary, to cell five. Further subdivisions are treated in a similar manner. However, when exercises are numbered at the margin and subdivisions thereof are column headings, as is often the case in arithmetic texts which are used in the lower grades, the tabular arrangement should, if possible, be preserved in the transcription. When the subdivisions of an exercise are so short that they can all be accommodated on one braille line, this should be done.

(1)



EXERCISES

1. Let $\alpha = (1, 1, 0), \beta = (-\frac{1}{2}, 0, \frac{2}{3}), \gamma = (0, \frac{1}{4}, 2)$.

Find

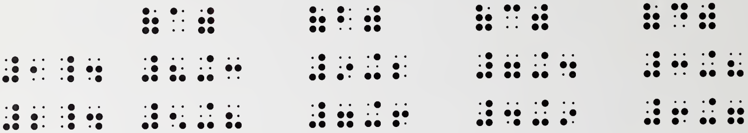
(a) $\alpha + 2\beta + 3\gamma$,

(b) $3(\alpha + \beta) - 2(\beta + \gamma)3$.

(c) What is the center of gravity of α, β, γ?

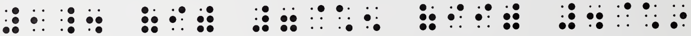
(d) Solve $6\beta + 5\xi = \alpha$.

(2)



	(a)	(b)	(c)	(d)
1.	$8 + 3$	$9 + 2$	$7 + 4$	$3 + 8$
2.	$5 + 8$	$7 + 6$	$4 + 9$	$6 + 7$

(3)

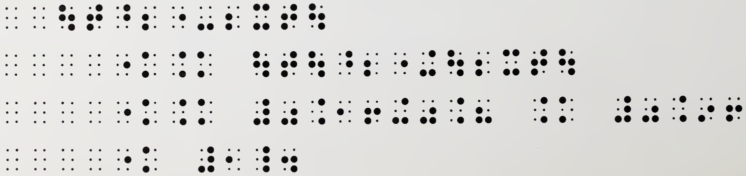


1. (i) 7×5 (ii) 4×9

d. When a mathematical expression *is* separated from the text by skipped lines, indentation, or some other means, the expression must begin in the third cell and the runover, if any, must begin in the fifth cell. This makes it easier for the reader to detect the beginning of the mathematical expression. If the mathematical expression is not arranged spatially, no lines should be skipped above or below it.

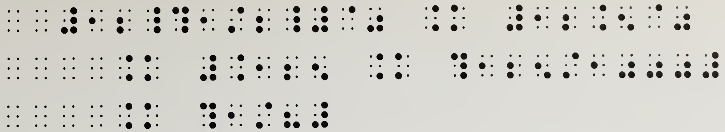
A *chain of mathematical expressions* is a mathematical expression whose several parts are joined by comparison signs, such as the equals sign, less than sign, greater than sign, inclusion or membership signs, or arrows. In such a chain, the first *link* extends from the beginning of the expression to the sign which precedes the first comparison sign; each additional link begins with a comparison sign and extends to the symbol which precedes the next comparison sign. A chain of mathematical expressions may contain as few as two links (one comparison sign). A chain of mathematical expressions must be written on one braille line, if possible. Otherwise, the first link must begin in cell three, and any runover must begin in cell five. Subsequent links must generally begin in cell five and any runover must then begin in cell seven. This provides for the vertical alignment of all the comparison symbols in the chain. However, if the links are very short, several of them may be placed on one braille line; in such an arrangement, no division between lines may be made within a link.

(1)



$$|t^2 - 2xt| \leqq |t|^2 + |2xt| \leqq 0.16 + 0.8 = 0.96 < 1.$$

(2)



$$12\tfrac{1}{2}\% = 12.5\% = .125 = \frac{125}{1000} = \frac{1}{8}$$

e. If a reference number or letter is associated with a mathematical expression set apart from the body of the text, this number or letter must begin in the third cell of a new braille line. In print, reference numbers or letters are sometimes at the right. In braille, these numbers or letters should be placed uniformly at the left. However, if reference numbers or letters occur at the right in print, a transcriber's note concerning the transposition of such numbers or letters must be included. The note should be placed at the beginning of the first volume. Page references which are associated with a mathematical expression should follow the expression.

f. When a mathematical expression must be divided at the end of a braille line, the division should be made giving priority to the following items in descending order:

Before a symbol of comparison

Before a symbol of operation or a fraction line

Before the base-line indicator or one of the opening fraction indicators

Before a change-of-level indicator or within a superscript before one of the symbols listed above

Between factors which are enclosed within grouping symbols

After the termination indicator

When a runover is made at a point of division listed in the first four items above, the symbol mentioned in that item must be the first on the new braille line.

g. The *beginning* of a superscript or subscript may never be disassociated from the symbol which carries it by the transition to another braille line. However, the division may be made, if necessary, *within* a superscript or subscript in accordance with the first four items in f. The transition to the new line of braille does not, of itself, terminate the effect of the superscript or subscript indicator (see §73g). If the transition is made within a superscript or subscript and before a comparison sign contained therein, or if the transition is made at the end of a superscript or subscript which is not followed by a comparison sign, the appropriate level indicator must be restated as the first symbol of the new braille line.

h. The argument associated with an abbreviated scientific word or phrase may never be separated from the abbreviated scientific word or phrase by transition to another braille line. However, if necessary, the transition may occur *within* the argument.

§185. Spacing with Spatial Arrangement: When the transcription is in the form of spatial arrangement, a blank line must be left both above and below the arrangement. Transition to a new braille page before beginning or after ending the transcription of a spatial arrangement takes the place of the required skipped line. The entire spatial arrangement should be confined to one braille page.

§186. Alignment of Fractions: When aligning fractions vertically, indicators should be placed under indicators, fraction lines under fraction lines, operation symbols under operation symbols, units digits under units digits, tens digits under tens digits, etc.

(1)

```
  ⠹⠂⠂⠌⠂⠖⠼      ⠹⠂⠌ ⠢⠼
 ⠤⠹ ⠂⠌ ⠆⠼     ⠤⠹⠂⠌⠂⠴⠼
⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒   ⠒⠒⠒⠒⠒⠒⠒⠒⠒
```

$$\begin{array}{r} \frac{11}{16} \\ -\ \frac{1}{2} \\ \hline \end{array} \qquad \begin{array}{r} \frac{1}{5} \\ -\ \frac{1}{10} \\ \hline \end{array}$$

APPENDIX

Note: The letters of these alphabets are those which will be found in mathematical texts.

§1. Combinations of Type-form, Alphabetic, and Capitalization Indicators:

LOWER-CASE LETTERS

Type-form	German letters	Greek letters	Hebrew letters	Roman letters	Russian letters
Boldface	⠸⠸	⠸⠨		⠸⠰	⠸⠈⠈
Italic	⠨⠸	⠨⠨		⠨⠰	⠨⠈⠈
Ordinary	⠀⠸	⠀⠨	⠀⠠⠠	⠀⠰	⠀⠈⠈
Sanserif					
Script	⠈⠸	⠈⠨	⠈⠠⠠	⠈⠰	⠈⠈⠈

CAPITALIZED LETTERS

Type-form	German letters	Greek letters	Hebrew letters	Roman letters	Russian letters
Boldface	⠸⠸⠠	⠸⠨⠠		⠸⠰⠠	⠸⠈⠈⠠
Italic	⠨⠸⠠	⠨⠨⠠		⠨⠰⠠	⠨⠈⠈⠠
Ordinary	⠀⠸⠠	⠀⠨⠠		⠀⠰⠠	⠀⠈⠈⠠
Sanserif				⠠⠫⠎	
Script	⠈⠸⠠	⠈⠨⠠		⠈⠰⠠	⠈⠈⠈⠠

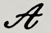
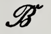
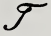
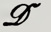
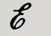
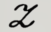
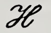
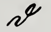
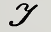
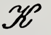
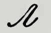
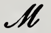
For the use of type-form indicators, alphabetic indicators, and capitalization indicators, see Rule VII.

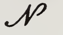
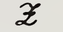
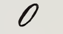
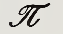
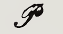
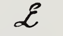
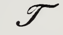
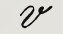
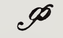
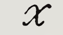
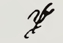
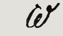
§2. German Alphabet:

Name of letter	Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Braille equivalent
ah	𝔞	𝔄			⠁
beh	𝔟	𝔅			⠃
tseh	𝔠	ℭ			⠉
deh	𝔡	𝔇			⠙
eh	𝔢	𝔈			⠑
eff	𝔣	𝔉			⠋
gheh	𝔤	𝔊			⠛
hah	𝔥	ℌ		or	⠓
ee	𝔦	ℑ			⠊
yaht	𝔧	ℑ		or	⠚
kah	𝔨	𝔎			⠅
ell	𝔩	𝔏			⠇
em	𝔪	𝔐			⠍
en	𝔫	𝔑			⠝
oh	𝔬	𝔒			⠕
peh	𝔭	𝔓			⠏
koo	𝔮	𝔔			⠟
err	𝔯	ℜ			⠗
ess	𝔰	𝔖	or		⠎
teh	𝔱	𝔗			⠞
oo	𝔲	𝔘			⠥
fao	𝔳	𝔙			⠧
veh	𝔴	𝔚			⠺
iks	𝔵	𝔛			⠭
ypsilon	𝔶	𝔜		or	⠽
tset	𝔷	ℨ			⠵

§3. Greek Alphabet:

a. Standard Alphabet:

Name of letter	Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Braille equivalent
alpha	α	Α			⠁
beta	β	Β			⠃
gamma	γ	Γ			⠛
delta	δ	Δ			⠙
epsilon	ϵ	Ε			⠑
zeta	ζ	Ζ			⠵
eta	η	Η			⠱
theta	θ	Θ			⠹
iota	ι	Ι			⠊
kappa	κ	Κ			⠅
lambda	λ	Λ			⠇
mu	μ	Μ			⠍

Name of letter	Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Braille equivalent
nu	ν	Ν			⠝
xi	ξ	Ξ			⠭
omicron	ο	Ο			⠕
pi	π	Π			⠏
rho	ρ	Ρ			⠗
sigma	σ	Σ			⠎
tau	τ	Τ			⠞
upsilon	υ	Υ			⠥
phi	ϕ	Φ			⠋
chi	χ	Χ			⠯
psi	ψ	Ψ			⠽
omega	ω	Ω			⠺

b. Alternative forms of Greek Letters:

Five of the Greek letters possess an alternative form in the ordinary lower-case alphabet. The braille equivalents are shown with their proper indicators. They are:

Name of letter	Ordinary lower-case	Braille equivalent
alpha	α	⠨⠈⠁
beta	ϐ	⠨⠈⠃
theta	ϑ	⠨⠈⠹
sigma	ς	⠨⠈⠎
phi	φ	⠨⠈⠋

§4. Hebrew Alphabet:

Hebrew letters do not possess a capitalized form.

Name of letter	Ordinary	Script	Braille equivalent	Name of letter	Ordinary	Script	Braille equivalent	Name of letter	Ordinary	Script	Braille equivalent
aleph	א	א	⠁	teth	ט	ט	⠞	feh	פ	פ	⠋
veth	ב	ב	⠧	yod	י	י	⠚	tsadi	צ	צ	⠮
gimel	ג	ג	⠛	chaph	כ	כ	⠡	koph	ק	ק	⠟
daleth	ד	ד	⠙	lamed	ל	ל	⠇	resh	ר	ר	⠗
heh	ה	ה	⠓	mem	מ	מ	⠍	sin	ש	ש	⠱
vav	ו	ו	⠺	nun	נ	נ	⠝	thav	ת	ת	⠹
zayin	ז	ז	⠵	samekh	ס	ס	⠎				
cheth	ח	ח	⠭	ayin	ע	ע	⠫				

§5. Roman Alphabet:

Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Sanserif capitalized	Braille equivalent
a	A	*a*	𝒜	A	⠁
b	B	*b*	ℬ	B	⠃
c	C	*c*	𝒞	C	⠉
d	D	*d*	𝒟	D	⠙
e	E	*e*	ℰ	E	⠑
f	F	*f*	ℱ	F	⠋
g	G	*g*	𝒢	G	⠛
h	H	*h*	ℋ	H	⠓
i	I	*i*	ℐ	I	⠊
j	J	*j*	𝒥	J	⠚
k	K	*k*	𝒦	K	⠅
l	L	*l*	ℒ	L	⠇
m	M	*m*	ℳ	M	⠍
n	N	*n*	𝒩	N	⠝
o	O	*o*	𝒪	O	⠕
p	P	*p*	𝒫	P	⠏
q	Q	*q*	𝒬	Q	⠟
r	R	*r*	ℛ	R	⠗
s	S	*s*	𝒮	S	⠎
t	T	*t*	𝒯	T	⠞

Roman Alphabet (continued)

Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Sanserif capitalized	Braille equivalent
u	U	*u*	*U*	U	⠥
v	V	*v*	*V*	V	⠧
w	W	*w*	*W*	W	⠺
x	X	*x*	*X*	X	⠭
y	Y	*y*	*Y*	Y	⠽
z	Z	*z*	*Z*	Z	⠵

§6. Russian Alphabet:

The Russian alphabet is sometimes referred to as the *Cyrillic alphabet.*

Name of letter	Ordinary lower-case	Ordinary capitalized	Script lower-case	Script capitalized	Braille equivalent
ah	а	А	*а*	*А*	⠁
beh	б	Б	*б*	*Б*	⠃
veh	в	В	*в*	*В*	⠺
gheh	г	Г	*г*	*Г*	⠛
deh	д	Д	*д* or *д*	*Д*	⠙
yeh	е	Е	*е*	*Е*	⠑
zheh	ж	Ж	*ж* or *ж*	*Ж*	⠚
zeh	з	З	*з* or *з*	*З*	⠵
ee	и	И	*и*	*И*	⠊
kah	к	К	*к* or *к*	*К*	⠅
ell	л	Л	*л*	*Л*	⠇
em	м	М	*м*	*М*	⠍
en	н	Н	*н*	*Н*	⠝
oh	о	О	*о*	*О*	⠕
peh	п	П	*п*	*П*	⠏
err	р	Р	*р*	*Р*	⠗
ess	с	С	*с*	*С*	⠎
teh	т	Т	*т* or *т̄*	*Т*	⠞
oo	у	У	*у*	*У*	⠥
eff	ф	Ф	*ф*	*Ф*	⠋
khah	х	Х	*х*	*Х*	⠓
tseh	ц	Ц	*ц*	*Ц*	⠉
cheh	ч	Ч	*ч*	*Ч*	⠟
shah	ш	Ш	*ш* or *ш̲*	*Ш*	⠱
shchah	щ	Щ	*щ*	*Щ*	⠭
yerih	ы	Ы	*ы*		⠮
eh	э	Э	*э*	*Э*	⠪
yu	ю	Ю	*ю*	*Ю*	⠳
yah	я	Я	*я*	*Я*	⠫

INDEX OF BRAILLE SYMBOLS

The following is the list of 63 braille symbols arranged in their standard order. The separation of these symbols into the usual seven lines of braille is ignored, but each symbol is numbered in accordance with its rank in the list and is identified by its dot numbers.

1	⠁	(dot 1)
2	⠃	(dots 1-2)
3	⠉	(dots 1-4)
4	⠙	(dots 1-4-5)
5	⠑	(dots 1-5)
6	⠋	(dots 1-2-4)
7	⠛	(dots 1-2-4-5)
8	⠓	(dots 1-2-5)
9	⠊	(dots 2-4)
10	⠚	(dots 2-4-5)
11	⠅	(dots 1-3)
12	⠇	(dots 1-2-3)
13	⠍	(dots 1-3-4)
14	⠝	(dots 1-3-4-5)
15	⠕	(dots 1-3-5)
16	⠏	(dots 1-2-3-4)
17	⠟	(dots 1-2-3-4-5)
18	⠗	(dots 1-2-3-5)
19	⠎	(dots 2-3-4)
20	⠞	(dots 2-3-4-5)
21	⠥	(dots 1-3-6)
22	⠧	(dots 1-2-3-6)
23	⠭	(dots 1-3-4-6)
24	⠽	(dots 1-3-4-5-6)
25	⠵	(dots 1-3-5-6)
26	⠯	(dots 1-2-3-4-6)
27	⠿	(dots 1-2-3-4-5-6)
28	⠷	(dots 1-2-3-5-6)
29	⠮	(dots 2-3-4-6)
30	⠾	(dots 2-3-4-5-6)
31	⠡	(dots 1-6)
32	⠣	(dots 1-2-6)
33	⠩	(dots 1-4-6)
34	⠹	(dots 1-4-5-6)
35	⠱	(dots 1-5-6)
36	⠫	(dots 1-2-4-6)
37	⠻	(dots 1-2-4-5-6)
38	⠳	(dots 1-2-5-6)
39	⠪	(dots 2-4-6)
40	⠺	(dots 2-4-5-6)
41	⠂	(dot 2)
42	⠆	(dots 2-3)
43	⠒	(dots 2-5)
44	⠲	(dots 2-5-6)
45	⠢	(dots 2-6)
46	⠖	(dots 2-3-5)
47	⠶	(dots 2-3-5-6)
48	⠦	(dots 2-3-6)
49	⠔	(dots 3-5)
50	⠴	(dots 3-5-6)
51	⠌	(dots 3-4)
52	⠬	(dots 3-4-6)
53	⠼	(dots 3-4-5-6)
54	⠜	(dots 3-4-5)
55	⠄	(dot 3)
56	⠤	(dots 3-6)
57	⠈	(dot 4)
58	⠘	(dots 4-5)
59	⠸	(dots 4-5-6)
60	⠐	(dot 5)
61	⠨	(dots 4-6)
62	⠰	(dots 5-6)
63	⠠	(dot 6)

The items in the INDEX OF BRAILLE SYMBOLS are "alphabetized" in accordance with the list of the 63 braille symbols above.

			Page
1	⠁ (dot 1)		
	⠁	German ah	See Appendix 126
		Greek alpha	See Appendix 127
		Hebrew aleph	See Appendix 128
		Roman a	See Appendix 128
		Russian ah	See Appendix 129
	⠁⠍⠏	amp, amplitude	103
	⠁⠝⠞⠊⠇⠕⠛	antilog, antilogarithm	103
	⠁⠗⠉	arc, arc	103
	⠁⠗⠛	arg, argument	103
2	⠃ (dots 1-2)		
	⠃	German beh	See Appendix 126
		Greek beta	See Appendix 127
		Roman b	See Appendix 128
		Russian beh	See Appendix 129
3	⠉ (dots 1-4)		
	⠉	German tseh	See Appendix 126
		Roman c	See Appendix 128
		Russian tseh	See Appendix 129

			Page
3	⠉ (Cont.)		
	⠉⠕⠇⠕⠛	colog, cologarithm	103
	⠉⠕⠎	cos, cosine	103
	⠉⠕⠎⠓	cosh, hyperbolic cosine	103
	⠉⠕⠞	cot, cotangent	103
	⠉⠕⠞⠓	coth, hyperbolic cotangent	103
	⠉⠕⠧⠑⠗⠎	covers, coversine	103
	⠉⠎⠉	csc, cosecant	103
	⠉⠎⠉⠓	csch, hyperbolic cosecant	103
	⠉⠞⠝	ctn, cotangent	103
	⠉⠞⠝⠓	ctnh, hyperbolic cotangent	103
4	⠙ (dots 1-4-5)		
	⠙	German deh	See Appendix 126
		Greek delta	See Appendix 127
		Roman d	See Appendix 128
		Russian deh	See Appendix 129
5	⠑ (dots 1-5)		
	⠑	German eh	See Appendix 126
		Greek epsilon	See Appendix 127
		Roman e	See Appendix 128
		Russian yeh	See Appendix 129

			Page
5	⠑ (Cont.)		
	⠑⠗⠋	erf, error function	103
	⠑⠭⠏	exp, exponential	103
	⠑⠭⠎⠑⠉	exsec, exsecant	103
6	⠋ (dots 1-2-4)		
	⠋	German eff	See Appendix 126
		Greek phi	See Appendix 127
		Hebrew feh	See Appendix 128
		Roman f	See Appendix 128
		Russian eff	See Appendix 129
7	⠛ (dots 1-2-4-5)		
	⠛	German gheh	See Appendix 126
		Greek gamma	See Appendix 127
		Hebrew gimel	See Appendix 128
		Roman g	See Appendix 128
		Russian gheh	See Appendix 129
8	⠓ (dots 1-2-5)		
	⠓	German hah	See Appendix 126
		Hebrew heh	See Appendix 128
		Roman h	See Appendix 128
		Russian khah	See Appendix 129
	⠓⠁⠧	hav, haversine	103

				Page
9	⠊	(dots 2-4)		
	⠊		German ee	See Appendix 126
			Greek iota	See Appendix 127
			Roman i	See Appendix 128
			Russian ee	See Appendix 129
	⠊⠝⠋		inf, infimum	103
10	⠚	(dots 2-4-5)		
	⠚		German yaht	See Appendix 126
			Hebrew yod	See Appendix 128
			Roman j	See Appendix 128
			Russian zheh	See Appendix 129
11	⠅	(dots 1-3)		
	⠅		German kah	See Appendix 126
			Greek kappa	See Appendix 127
			Roman k	See Appendix 128
			Russian kah	See Appendix 129
12	⠇	(dots 1-2-3)		
	⠇		German ell	See Appendix 126
			Greek lambda	See Appendix 127
			Hebrew lamed	See Appendix 128

			Page
12	⠇ (Cont.)		
		Roman l	See Appendix 128
		Russian ell	See Appendix 129
	⠇⠊⠍	lim, limit	104
	⠇⠝	ln, natural logarithm	104
	⠇⠕⠛	log, logarithm	104
13	⠍ (dots 1-3-4)		
	⠍	German em	See Appendix 126
		Greek mu	See Appendix 127
		Hebrew mem	See Appendix 128
		Roman m	See Appendix 128
		Russian em	See Appendix 129
	⠍⠁⠭	max, maximum	104
	⠍⠊⠝	min, minimum	104
	⠍⠕⠙	mod, modulo	104
14	⠝ (dots 1-3-4-5)		
	⠝	German en	See Appendix 126
		Greek nu	See Appendix 127
		Hebrew nun	See Appendix 128
		Roman n	See Appendix 128
		Russian en	See Appendix 129

			Page
15	⠕	(dots 1-3-5)	
	⠕	Barbed right full arrowhead	88
		German oh	See Appendix 126
		Greek omicron	See Appendix 127
		Roman o	See Appendix 128
		Russian oh	See Appendix 129
	⠕ ⠒⠒⠒	inverted division format with curved side	107
16	⠏	(dots 1-2-3-4)	
	⠏	German peh	See Appendix 126
		Greek pi	See Appendix 127
		Roman p	See Appendix 128
		Russian peh	See Appendix 129
17	⠟	(dots 1-2-3-4-5)	
	⠟	German koo	See Appendix 126
		Hebrew koph	See Appendix 128
		Roman q	See Appendix 128
		Russian cheh	See Appendix 129
18	⠗	(dots 1-2-3-5)	
	⠗	German err	See Appendix 126
		Greek rho	See Appendix 127

			Page
18	⠗ (Cont.)		
		Hebrew resh	See Appendix 128
		Roman r	See Appendix 128
		Russian err	See Appendix 129
19	⠎ (dots 2-3-4)		
	⠎	German ess	See Appendix 126
		Greek sigma	See Appendix 127
		Hebrew samekh	See Appendix 128
		Roman s	See Appendix 128
		Russian ess	See Appendix 129
	⠎ ⠑ ⠉	sec, secant	104
	⠎ ⠑ ⠉ ⠓	sech, hyperbolic secant	104
	⠎ ⠊ ⠝	sin, sine	104
	⠎ ⠊ ⠝ ⠓	sinh, hyperbolic sine	104
	⠎ ⠥ ⠏	sup, supremum	104
20	⠞ (dots 2-3-4-5)		
	⠞	German teh	See Appendix 126
		Greek tau	See Appendix 127
		Hebrew teth	See Appendix 128
		Roman t	See Appendix 128
		Russian teh	See Appendix 129
	⠞ ⠁ ⠝	tan, tangent	104
	⠞ ⠁ ⠝ ⠓	tanh, hyperbolic tangent	104

				Page
21	⠥	(dots 1-3-6)		
	⠥		German oo	See Appendix 126
			Greek upsilon	See Appendix 127
			Roman u	See Appendix 129
			Russian oo	See Appendix 129
22	⠧	(dots 1-2-3-6)		
	⠧		German fao	See Appendix 126
			Hebrew veth	See Appendix 127
			Roman v	See Appendix 129
	⠧⠑⠗⠎		vers, versine	104
23	⠭	(dots 1-3-4-6)		
	⠭		German iks	See Appendix 126
			Greek xi	See Appendix 127
			Hebrew cheth	See Appendix 128
			Roman x	See Appendix 129
			Russian shchah	See Appendix 129
24	⠽	(dots 1-3-4-5-6)		
	⠽		curved right full arrowhead	) 89
			German ypsilon	See Appendix 126
			Greek psi	See Appendix 127
			Roman y	See Appendix 129

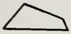
					Page
25	⠵	(dots 1-3-5-6)			
	⠵		German tset		See Appendix 126
			Greek zeta		See Appendix 127
			Hebrew zayin		See Appendix 128
			Roman z		See Appendix 129
			Russian zeh		See Appendix 129
26	⠯	(dots 1-2-3-4-6)			
	⠯		curved left full arrowhead	(	89
			factorial	!	98
			Greek chi		See Appendix 127
27	⠿	(dots 1-2-3-4-5-6)			
	⠿		blunted left full arrowhead	[	88
			blunted right full arrowhead	]	89
			omission indicator		23
28	⠷	(dots 1-2-3-5-6)			
	⠷		left parenthesis	(	95
	⠷⠈⠫⠢⠔⠕		left parenthesis superposed on counterclockwise arrow		82
29	⠮	(dots 2-3-4-6)			
	⠮		Hebrew tsade		See Appendix 128
			integral	∫	98
			Russian yerih		See Appendix 129

				Page
29	⠮ (Cont.)			
	⠮⠮	double integral	∬	98
	⠮⠮⠮	triple integral	∭	98
	⠮⠈⠫⠉	integral with superposed circle	∮	98
	⠮⠈⠫⠗	integral with superposed rectangle		98
	⠮⠈⠫⠲	integral with superposed square		99
	⠮⠈⠠⠿	integral with superposed infinity		98
30	⠾ (dots 2-3-4-5-6)			
	⠾	right parenthesis	)	95
	⠾⠈⠈⠷	interlocking parentheses	≬	82
31	⠡ (dots 1-6)			
	⠡	dot	·	45, 70
		Hebrew chaph	See Appendix	128
		oblique negation sign (upper left to lower right)	\	75
	⠡⠈⠸⠐⠅	dot within inclusion sign	⪽	81
	⠡⠈⠸⠨⠂	dot within reverse inclusion sign	⪾	81
	⠡⠈⠨⠅	dot between bars of equals sign		81
32	⠣ (dots 1-2-6)			
	⠣	directly-over indicator		44
		index-of-radical indicator		54
		makes nearer arrowhead point north		88

				Page
32	⠣ (Cont.)			
	⠣⠇⠊⠍	upper limit	$\overline{\lim}$	104
	⠣⠮	upper integral	$\bar{\int}$	98
33	⠩ (dots 1-4-6)			
	⠩	directly-under indicator		44
		makes nearer arrowhead point south		88
	⠩⠇⠊⠍	lower limit	$\underline{\lim}$	104
	⠩⠮	lower integral	$\underline{\int}$	98
34	⠹ (dots 1-4-5-6)			
	⠹	Greek theta	See Appendix	127
		Hebrew thav	See Appendix	128
		opening simple fraction indicator		15
35	⠱ (dots 1-5-6)			
	⠱	Greek eta	See Appendix	127
		Hebrew sin	See Appendix	128
		horizontal bar (macron)	—	45, 75
		Russian shah	See Appendix	129
	⠱⠈⠩	bar over logical product	$\overline{\wedge}$	79
	⠱⠈⠩⠱	bar over and bar under logical product	$\underline{\overline{\wedge}}$	79
	⠱⠈⠩⠨⠅	bar over and equals sign under logical product	$\underset{=}{\overline{\wedge}}$	79
	⠱⠈⠱	bar over single tilde	$\overline{\sim}$	80
	⠱⠈⠱⠈⠱	bar over double tilde	$\overline{\approx}$	80

				Page
35	⠱ (Cont.)			
	⠱⠈⠬	bar over logical sum	$\overline{\vee}$	79
	⠱⠈⠬⠱	bar over and bar under logical sum	$\underline{\overline{\vee}}$	79
	⠱⠈⠬⠨⠅	bar over and equals sign under logical sum	$\underset{=}{\overline{\vee}}$	79
	⠱⠈⠸⠐⠅	bar through inclusion sign	⋲	82
	⠱⠈⠸⠨⠂	bar through reverse inclusion sign	⋺	82
	⠱⠸⠐⠅	bar over inclusion sign	$\overline{\subset}$	78
	⠱⠸⠨⠂	bar over reverse inclusion sign	$\overline{\supset}$	80
	⠱⠐⠅	bar over less than sign	$\overline{<}$ or $\leqslant$	78
	⠱⠨⠂	bar over greater than sign	$\overline{>}$ or $\geqslant$	78
36	⠫ (dots 1-2-4-6)			
	⠫	Hebrew ayin	See Appendix	128
		Russian yah	See Appendix	129
		shape indicator		58
	⠫⠁	arc opening downward	⌒	44, 58, 75
	⠫⠉	circle	○	58
	⠫⠉⠈⠱	circle with superposed horizontal bar	⊖	62
	⠫⠉⠈⠳	circle with superposed vertical bar	⦶	62
	⠫⠉⠨⠎	small circle	$\circ$	60
	⠫⠉⠨⠡	circle with interior dot	⊙	61, 69
	⠫⠉⠨⠫⠣⠒⠒⠕	circle with interior arrow north	$\textcircled{\uparrow}$	61
	⠫⠉⠨⠫⠣⠒⠒⠕⠐⠫⠩⠫⠒⠒⠕	circle with interior arrow north followed by interior arrow south	$\textcircled{\uparrow\downarrow}$	61

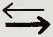
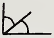
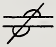
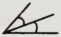
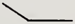
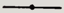
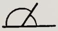
36 ⠫ (Cont.)

			Page
⠫⠉⠨⠫⠩⠒⠒⠕	circle with interior arrow south		61
⠫⠉⠨⠫⠩⠒⠒⠕⠐⠫⠣⠒⠒⠕	circle with interior arrow south followed by interior arrow north		61
⠫⠉⠨⠫⠪⠒⠒	circle with interior arrow west		60
⠫⠉⠨⠫⠪⠒⠒⠫⠒⠒⠕	circle with interior arrow west over interior arrow east		61
⠫⠉⠨⠫⠒⠒⠕	circle with interior arrow east		60
⠫⠉⠨⠫⠒⠒⠕⠫⠪⠒⠒	circle with interior arrow east over interior arrow west		61
⠫⠉⠨⠳⠐⠱⠐⠳	circle with interior horizontal bar between two vertical bars		61
⠫⠉⠨⠬	circle with interior plus sign	⊕	61, 69
⠫⠉⠨⠤	circle with interior minus sign	⊖	61
⠫⠉⠨⠈⠡	circle with interior cross	⊗	61
⠫⠉⠨⠠⠓	circle with interior capitalized h	Ⓗ	61
⠫⠙	diamond	◇	58
⠫⠑	ellipse	⬭	58
⠫⠛	parallelogram	▱	59
⠫⠓	rhombus	▱	59
⠫⠓⠭	irregular hexagon	⬡	58
⠫⠊	intersecting lines	×	59
⠫⠇	is parallel to ✓	∥	59
⠫⠏	is perpendicular to ✓	⊥	59
	northwest corner	┌	62
⠫⠏⠛	irregular pentagon	⬠	59
⠫⠟	quadrilateral		59

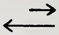
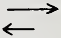
36 ⠫ (Cont.)

			Page
⠫⠗	rectangle	▭	59
⠫⠗⠭	prescription sign	℞	99
⠫⠎	sanserif-type indicator		See Appendix 125, 26
⠫⠞	triangle	△	59
⠫⠞⠨⠁	acute triangle	△	60
⠫⠞⠨⠊	isosceles triangle	△	60
⠫⠞⠨⠕	obtuse triangle	△	60
⠫⠞⠨⠗	right triangle	◺	60
⠫⠧	factorial	∟	62
	southwest corner	∟	62
⠫⠵	trapezoid	⏢	59
⠫⠡⠒⠒	arrow dotted at left	•——	45
⠫⠡⠒⠒⠕	arrow dotted at left and barbed at right	•⟶	45
⠫⠡⠒⠒⠡	arrow dotted at both ends	•——•	45
⠫⠣⠒⠒⠕⠐⠫⠩⠒⠒⠕	arrow north followed by arrow south	↑↓	80
⠫⠣⠒⠒⠕⠐⠫⠩⠸⠒⠒⠕	arrow north followed by boldface arrow south	↑**↓**	81
⠫⠣⠸⠒⠒⠕⠐⠫⠩⠒⠒⠕	boldface arrow north followed by arrow south	**↑**↓	81
⠫⠣⠸⠒⠒⠕⠐⠫⠩⠸⠒⠒⠕	boldface arrow north followed by boldface arrow south	**↑↓**	81
⠫⠩⠒⠒⠕⠐⠫⠣⠒⠒⠕	arrow south followed by arrow north	↓↑	80
⠫⠩⠒⠒⠕⠐⠫⠣⠸⠒⠒⠕	arrow south followed by boldface arrow north	↓**↑**	81
⠫⠩⠸⠒⠒⠕⠐⠫⠣⠒⠒⠕	boldface arrow south followed by arrow north	**↓**↑	81
⠫⠩⠸⠒⠒⠕⠐⠫⠣⠸⠒⠒⠕	boldface arrow south followed by boldface arrow north	**↓↑**	81

36 ⠫ (Cont.)

			Page
⠫⠹	northeast corner	┐	62
⠫⠪	angle	∠	58
⠫⠪⠒⠒	arrow barbed at left	←	44
	left-pointing arrow	←	75
⠫⠪⠒⠒⠕	arrow barbed at both ends	↔	44
	two-way arrow	↔	75
⠫⠪⠒⠒⠡	arrow barbed at left and dotted at right	←•	45
⠫⠪⠒⠒⠫⠸⠒⠒⠕	arrow west over boldface arrow east		77
⠫⠪⠶⠶⠕⠐⠸⠇⠐⠈⠱	spear west-east followed by identity sign followed by tilde	⇔ ≡ ∼	81
⠫⠪⠈⠫⠁	angle with superposed arc		62
⠫⠪⠨⠁	acute angle	∠	59
⠫⠪⠨⠁⠑	alternate exterior angles		59
⠫⠪⠨⠁⠊	alternate interior angles		59
⠫⠪⠨⠉	corresponding angles		59
⠫⠪⠨⠉⠏	complementary angles		59
⠫⠪⠨⠑	exterior angles		59
⠫⠪⠨⠊	interior angles		59
⠫⠪⠨⠚	adjacent angles		59
⠫⠪⠨⠕	obtuse angle		60
⠫⠪⠨⠗	right angle	∟	60
⠫⠪⠨⠎	straight angle		60
⠫⠪⠨⠎⠏	supplementary angles		60
⠫⠪⠨⠧	vertical angles		60

36 ⠫ (Cont.)

			Page
⠫⠪⠨⠫⠁	angle with interior arc	∡	60
⠫⠪⠨⠫⠪⠢⠔	angle with interior clockwise arrow		60
⠫⠪⠨⠫⠢⠔⠕	angle with interior counterclockwise arrow		60
⠫⠒	equilateral triangle	△	45, 58
⠫⠒⠕⠫⠪⠒⠒⠒	short arrow east over long arrow west		78
⠫⠒⠒⠅	difference between		75
⠫⠒⠒⠕	arrow barbed at right	→	45
	right-pointing arrow	→	75
⠫⠒⠒⠕⠫⠪⠒⠒	arrow east over arrow west	⇄	77
⠫⠒⠒⠕⠫⠸⠪⠒⠒	arrow east over boldface arrow west		77
⠫⠒⠒⠡	arrow dotted at right		45
⠫⠒⠒⠪	is a part of		75
⠫⠒⠒⠒⠕⠫⠪⠒	long arrow east over short arrow west		78
⠫⠒⠒⠈⠕⠫⠠⠪⠒⠒	arrow with upper barb east over arrow with lower barb west	⇌	77
⠫⠲	square	□	59, 70
⠫⠲⠈⠱	square with superposed horizontal bar		62
⠫⠲⠈⠳	square with superposed vertical bar		62
⠫⠲⠨⠡	square with interior dot	⊡	61
⠫⠲⠨⠱	square with interior horizontal bar	⊟	61
⠫⠲⠨⠳	square with interior vertical bar	◫	61
⠫⠲⠨⠢	square with interior northwest-southeast diagonal	⧅	62
⠫⠲⠨⠢⠈⠔	square with interior diagonals	⊠	61

				Page
36	⠫ (Cont.)			
	⠫⠲⠨⠔	square with interior southwest-northeast diagonal	⧄	62
	⠫⠢	regular pentagon	⬠	59
	⠫⠖	regular hexagon	⬣	58
	⠫⠼	southeast corner	┘	62
	⠫⠄	arc opening upward	◡	44, 58, 75
	⠫⠸⠪⠒⠒⠫⠒⠒⠕	boldface arrow west over arrow east	⇆	77
	⠫⠸⠪⠒⠒⠫⠸⠒⠒⠕	boldface arrow west over boldface arrow east	⇆	78
	⠫⠸⠒⠒⠕⠫⠪⠒⠒	boldface arrow east over arrow west	⇄	77
	⠫⠸⠒⠒⠕⠫⠸⠪⠒⠒	boldface arrow east over boldface arrow west	⇄	77
	⠫⠸⠲	filled-in square	■	69
37	⠻ (dots 1-2-4-5-6)			
	⠻	closing cancellation indicator		24
		termination indicator		54
38	⠳ (dots 1-2-5-6)			
	⠳	Russian yu	See Appendix	129
		straight left full arrowhead	├	89
		straight right full arrowhead	┤	89
		vertical bar	|	45, 95, 99
		vertical negation line	|	75
	⠳⠱⠈⠱	is divisible by		87
	⠳⠳	double vertical bar	‖	95

					Page
38	⠳	(Cont.)			
	⠳		inverted division format with straight side	└	107
	⠒⠒⠒				
	⠳⠈⠫⠪⠒⠒		vertical bar through shaft of arrow west	⇷	82
	⠳⠈⠫⠒⠒⠕		vertical bar through shaft of arrow east	⇸	82
39	⠪	(dots 2-4-6)			
	⠪		barbed left full arrowhead	‹	88
			opening cancellation indicator		24
			Russian eh	See Appendix	129
40	⠺	(dots 2-4-5-6)			
	⠺		German veh	See Appendix	126
			Greek omega	See Appendix	127
			Hebrew vav	See Appendix	128
			Roman w	See Appendix	129
			Russian veh	See Appendix	129
41	⠂	(dot 2)			
	⠂		numeral 1	1	1
42	⠆	(dots 2-3)			
	⠆		numeral 2	2	1
			semicolon	;	10
43	⠒	(dots 2-5)			
	⠒		colon	:	9
			numeral 3	3	1

				Page
43	⠒ (Cont.)			
		short single arrow shaft	—	88
	⠒⠒	ordinary single arrow shaft	——	88
	⠒⠒⠒	long single arrow shaft	———	88
		separation line	————	107
	⠒⠒⠒⠒⠒	fraction line in spatial arrangement	——	16
	⠒⠒⠒ ⠕	normal division format with curved side	$)\overline{\quad\quad}$	107 ✓
	⠒⠒⠒ ⠳	normal division format with straight side	$\lceil\overline{\quad\quad}$	107 ✓
	⠒⠒⠒ ⠜	square root format	$\sqrt{\quad\quad}$	107
44	⠲ (dots 2-5-6)			
	⠲	numeral 4	4	1
		period	.	9
45	⠢ (dots 2-6)			
	⠢	numeral 5	5	1
	⠢⠔	curved arrow shaft	⌒	88
46	⠖ (dots 2-3-5)			
	⠖	exclamation point	!	9
		numeral 6	6	1

					Page
47	⠶	(dots 2-3-5-6)			
	⠶		left literary parenthesis	(	9
			numeral 7	7	1
			right literary parenthesis	)	9
			short double arrow shaft	=	88
	⠶⠆		right literary brace	}	9
	⠶⠶		ordinary double arrow shaft	==	88
	⠶⠶⠶		long double arrow shaft	===	88
			separation line (for changing number form, carrying or borrowing)	———	107
	⠶⠄		right literary bracket	]	9
48	⠦	(dots 2-3-6)			
	⠦		left outer quotation mark	“	10
			numeral 8	8	1
			question mark	?	9
49	⠔	(dots 3-5)			
	⠔		numeral 9	9	1
50	⠴	(dots 3-5-6)			
	⠴		numeral 0	0	1
			right outer quotation mark	”	10
	⠴⠄		right inner quotation mark	’	10
51	⠌	(dots 3-4)			
	⠌		oblique negation sign (lower left to upper right)	/	75

					Page
51	⠌	(Cont.)			
			simple fraction line	—	16, 70 ✓
	⠌⠈⠱⠈⠱		of the form of	≉	87
	⠌⠠⠡		it does not follow that	/∴	102
52	⠬	(dots 3-4-6)			
	⠬		plus	+	70 ✓
	⠬⠤		plus or minus	±	70
	⠬⠐⠤		plus followed by minus	+ —	70
53	⠼	(dots 3-4-5-6)			
	⠼		closing simple-fraction indicator		15
			numeric indicator		1
54	⠜	(dots 3-4-5)			
	⠜		radical	$\sqrt{\quad}$	54
55	⠄	(dot 3)			
	⠄		apostrophe	’	8
			prime	′	99
56	⠤	(dots 3-6)			
	⠤		hyphen	-	9
			minus	—	70 ✓
	⠤⠬		minus or plus	∓	70
	⠤⠤		short dash	—	9
	⠤⠤⠤⠤		long dash	——	9
	⠤⠐⠬		minus followed by plus	— +	70

				Page
57	⠈	(dot 4)		
	⠈	script-type indicator		See Appendix 125, 26
		superposition indicator		58
	⠈⠁	at	@	98 ✓
	⠈⠉	cent	¢	98 ✓
	⠈⠙	round d, partial derivative sign	∂	99
	⠈⠑	membership, is an element of	ε	76
	⠈⠓	crossed h	ħ	98
	⠈⠇	pound sterling	£	99
	⠈⠕	barbed right upper arrowhead	⇀	88
	⠈⠏	Weierstrass p	℘	99
	⠈⠎	dollar sign	$	98 ✓
	⠈⠽	curved right upper arrowhead	⌒	89
	⠈⠯	curved left upper arrowhead	(	89
		universal quantifier, for all, for each	∀ ∀	99
	⠈⠿	blunted left upper arrowhead	⌈	88
		blunted right upper arrowhead	⌉	89
		existential quantifier, there exists, for some	∃	98
	⠈⠿⠳	there exists uniquely	∃|	100
	⠈⠷	left bracket	[	95
	⠈⠾	right bracket	]	95
	⠈⠡	cross, multiplication sign	×	70 ✓

57 ⠈ (Cont.)

			Page
⠈⠣	pipestrap	⌢	76
⠈⠣⠱	difference between	≏	78
⠈⠣⠠⠣	equivalence	≍	78
⠈⠩	logical product (and, meet)	∧	70
⠈⠩⠱	bar under logical product	⩟	79
⠈⠩⠨⠅	equals sign under logical product	⩠	79
⠈⠱	single tilde	∼	45, 76
⠈⠱⠱	bar under single tilde	≃	80
⠈⠱⠈⠱	double tilde	≈	80
⠈⠱⠈⠱⠱	bar under double tilde	≊	80
⠈⠱⠈⠱⠨⠅	equals sign under double tilde	$\underline{\underline{\approx}}$	80
⠈⠱⠨⠅	equals sign under single tilde	≅	80
⠈⠫	crossed d	đ	98
⠈⠳	straight left upper arrowhead	⌊	89
	straight right upper arrowhead	⌋	89
⠈⠪	barbed left upper arrowhead	∠	88
⠈⠢	reverse membership sign	∋	76
⠈⠴	percent sign	%	99
⠈⠌	since	∵	99
⠈⠬	logical sum	∨	70
⠈⠬⠱	bar under logical sum	⊻	79
⠈⠬⠨⠅	equals sign under logical sum	⩣	79

				Page
57	⠈ (Cont.)			
	⠈⠼	asterisk	*	8, 69 ✓
	⠈⠜	check mark	√	9
	⠈⠈	Russian-letter indicator	See Appendix	125, 26
	⠈⠈⠈	lower-case script Russian-letter indicator	See Appendix	125
	⠈⠈⠈⠠	capital script Russian-letter indicator	See Appendix	125
	⠈⠸	lower-case script German-letter indicator	See Appendix	125
	⠈⠸⠷	barred left bracket	⟦	94
	⠈⠸⠾	barred right bracket	⟧	94
	⠈⠸⠠	capital script German-letter indicator	See Appendix	125
	⠈⠨	lower-case script Greek-letter indicator	See Appendix	125
	⠈⠨⠠	capital script Greek-letter indicator	See Appendix	125
	⠈⠰	lower-case script Roman-letter indicator	See Appendix	125
	⠈⠰⠠	capital script Roman-letter indicator	See Appendix	125
	⠈⠠⠁	angstrom unit	Å	98
	⠈⠠⠏	paragraph mark	¶	9, 70
	⠈⠠⠎	section mark	§	10, 70
	⠈⠠⠷	enlarged left bracket	[	56
	⠈⠠⠾	enlarged right bracket	]	56
	⠈⠠⠱	extended tilde	∼	45, 76
	⠈⠠⠠	lower-case script Hebrew-letter indicator	See Appendix	125
58	⠘ (dots 4-5)			
	⠘	elevates nearer arrowhead by 45 degrees		87

58 ⠘ (Cont.)

			Page
	superscript indicator		34
⠘⠘	superscript to superscript indicator		34
⠘⠘⠘	superscript to superscript to superscript indicator		35
⠘⠘⠰	subscript to superscript to superscript indicator		35
⠘⠰	subscript to superscript indicator		34
⠘⠰⠘	superscript to subscript to superscript indicator		35
⠘⠰⠰	subscript to subscript to superscript indicator		35

59 ⠸ (dots 4-5-6)

			Page
⠸	boldface-type indicator	See Appendix	125, 26, 88
	German-letter indicator	See Appendix	125, 26
	punctuation indicator		8
	tally mark	|	10
⠸⠇	is congruent to, is identical with	≡	75
⠸⠿	variation sign, varies as	∝	76
⠸⠡	filled-in dot	•	69
⠸⠣	caret, circumflex	^	45, 98
⠸⠩	inverted caret	∨	45
⠸⠹	opening fractional-part-of-mixed-number indicator		15
⠸⠳	single boldface vertical bar	**|**	95
⠸⠳⠸⠳	double boldface vertical bar	**‖**	95
⠸⠦	question mark (as modifier)	?	45
⠸⠴	empty set	∅ or ϕ	98

				Page
59	⠸ (Cont.)			
	⠸⠌	diagonal line	/	16, 70
	⠸⠼	closing fractional-part-of-mixed-number indicator		15
	⠸⠈⠷	boldface left bracket	**[**	95
	⠸⠈⠾	boldface right bracket	**]**	95
	⠸⠈⠈	lower-case boldface Russian-letter indicator	See Appendix	125
	⠸⠈⠈⠠	capital boldface Russian-letter indicator	See Appendix	125
	⠸⠸	lower-case boldface German-letter indicator	See Appendix	125
	⠸⠸⠠	capital boldface German-letter indicator	See Appendix	125
	⠸⠐⠅	inclusion sign (is contained in)	$\subset$	75
	⠸⠐⠅⠱	bar under inclusion sign	$\subseteq$	78
	⠸⠐⠅⠨⠅	equals sign under inclusion sign	$\subseteqq$	78
	⠸⠨	lower-case boldface Greek-letter indicator	See Appendix	125
	⠸⠨⠅	boldface equals sign	**=**	75
	⠸⠨⠂	reverse inclusion sign (contains)	$\supset$	75
	⠸⠨⠂⠱	bar under reverse inclusion sign	$\supseteq$	80
	⠸⠨⠂⠨⠅	equals sign under reverse inclusion sign	$\supseteqq$	80
	⠸⠨⠠	capital boldface Greek-letter indicator	See Appendix	125
	⠸⠰	lower-case boldface Roman-letter indicator	See Appendix	125
	⠸⠰⠠	capital boldface Roman-letter indicator	See Appendix	125
	⠸⠠	capital German-letter indicator	See Appendix	125
60	⠐ (dot 5)			
	⠐	base-line indicator		34

60 ⠐ (Cont.)			Page
	multipurpose indicator		53
⠐⠅	less than sign	$<$	76
⠐⠅⠱	bar under less than sign	$\leq$ or $\leqslant$	79
⠐⠅⠈⠐⠅	nest of two less than signs (is small compared with)	$\ll$	82
⠐⠅⠐⠨⠅⠐⠨⠂	less than sign followed by equals sign followed by greater than sign	$< = >$	81
⠐⠅⠐⠨⠂	less than sign followed by greater than sign	$< >$	81
⠐⠅⠨⠅	less than or equal to	$\leqq$	79
⠐⠱⠣⠡⠻	dot over horizontal bar	$\overset{\cdot}{-}$	77
⠐⠱⠩⠸⠣⠻	caret under horizontal bar	$\overline{\wedge}$	77
⠐⠂	ratio sign, is to	:	76
⠐⠈⠱⠩⠡⠻	dot under simple tilde	$\underset{\cdot}{\sim}$	77
⠐⠨⠅⠣⠡⠻	dot over equals sign	$\doteq$	76
⠐⠨⠅⠣⠫⠒⠻	equilateral triangle over equals sign	$\triangleq$	76
⠐⠨⠅⠣⠳⠻	vertical bar over equals sign	$\overset{\vert}{=}$	77
⠐⠨⠅⠣⠸⠣⠻	caret over equals sign	$\overset{\wedge}{=}$	76
⠐⠨⠅⠣⠸⠩⠻	inverted caret over equals sign	$\overset{\vee}{=}$	77
⠐⠨⠅⠣⠸⠦⠻	question mark over equals sign	$\overset{?}{=}$	77
⠐⠨⠅⠣⠨⠡⠻	degree sign over equals sign	$\overset{\circ}{=}$	76
⠐⠨⠅⠣⠰⠣⠻	left-pointing caret over equals sign	$\overset{<}{=}$	77
⠐⠨⠅⠣⠰⠩⠻	right-pointing caret over equals sign	$\overset{>}{=}$	77
⠐⠨⠅⠩⠡⠡⠣⠡⠡⠻	two dots over and two dots under equals sign	$\underset{\cdot\,\cdot}{\overset{\cdot\,\cdot}{=}}$	77

				Page
60	⠐ (Cont.)			
	⠐⠨⠅⠩⠡⠣⠡⠻	dot over and dot under equals sign	$\overset{\cdot}{\underset{\cdot}{=}}$	76
	⠐⠨⠅⠩⠸⠣⠻	caret under equals sign	$\underset{\wedge}{=}$	76
61	⠨ (dots 4-6)			
	⠨	decimal point	.	98 ✓
		first inner radical indicator		54
		Greek-letter indicator	See Appendix	125, 26
		italic-type indicator	See Appendix	125, 26
		shape-modification indicator		58
	⠨⠉	is greater than	$\sqsubset$	75
	⠨⠅	equals sign, is equal to	$=$	75 ✓
	⠨⠅⠈⠩	equals sign over logical product	$\overset{=}{\wedge}$	79
	⠨⠅⠈⠩⠱	equals sign over and bar under logical product	$\overset{=}{\underset{-}{\wedge}}$	79
	⠨⠅⠈⠩⠨⠅	equals sign over and equals sign under logical product	$\overset{=}{\underset{=}{\wedge}}$	79
	⠨⠅⠈⠱	equals sign over single tilde	$\overset{=}{\sim}$	80
	⠨⠅⠈⠱⠈⠱	equals sign over double tilde	$\overset{=}{\approx}$	80
	⠨⠅⠈⠬	equals sign over logical sum	$\overset{=}{\vee}$	79
	⠨⠅⠈⠬⠱	equals sign over and bar under logical sum	$\overset{=}{\underset{-}{\vee}}$	79
	⠨⠅⠈⠬⠨⠅	equals sign over and equals sign under logical sum	$\overset{=}{\underset{=}{\vee}}$	79
	⠨⠅⠈⠸⠐⠅	equals sign through inclusion sign	$=\!\!\!\!\subset$	82
	⠨⠅⠈⠸⠨⠂	equals sign through reverse inclusion sign	$\supset\!\!\!\!=$	82
	⠨⠅⠸⠐⠅	equals sign over inclusion sign	$\overset{=}{\subset}$	78

61 ⠨ (Cont.)

			Page
⠨⠅⠸⠨⠂	equals sign over reverse inclusion sign	$\overset{=}{\supset}$	80
⠨⠅⠐⠅	is equal to or less than	$\overset{=}{<}$	79
⠨⠅⠨⠂	is equal to or greater than	$\overset{=}{>}$	78
⠨⠷	left brace	{	94
⠨⠾	right brace	}	94
⠨⠡	degree sign	°	70
	hollow dot	∘	45
⠨⠩	intersection sign (cap)	∩	70
⠨⠩⠱	bar under intersection sign	$\underline{\cap}$	78
⠨⠩⠨⠅	equals sign under intersection sign	$\underset{=}{\cap}$	78
⠨⠫	del, nabla	∇	98
⠨⠳	dagger	†	9
⠨⠂	greater than sign	>	75
⠨⠂⠱	bar under greater than sign	$\underline{>}$ or ⩾	78
⠨⠂⠈⠨⠂	nest of two greater than signs	≫	82
⠨⠂~~⠐~~⠐⠅	greater than sign followed by less than sign	> <	81
⠨⠂⠐⠨⠅⠐⠐⠅	greater than sign followed by equals sign followed by less than sign	> = <	81
⠨⠂⠨⠅	greater than or equal to	$\underset{=}{>}$	78
⠨⠌	division sign	÷	69
⠨⠬	union sign (cup)	∪	70
⠨⠬⠱	bar under union sign	$\underline{\cup}$	80
⠨⠬⠨⠅	equals sign under union sign	$\underset{=}{\cup}$	80

61 ⠨ (Cont.)			Page
⠨⠼	number sign	#	70 ✓
⠨⠤	is less than	⌝	76
⠨⠈⠈	lower-case italic Russian-letter indicator	See Appendix	125
⠨⠈⠈⠠	capital italic Russian-letter indicator	See Appendix	125
⠨⠸	lower-case italic German-letter indicator	See Appendix	125
⠨⠸⠷	barred left brace	⦃	94
⠨⠸⠾	barred right brace	⦄	94
⠨⠸⠠	capital italic German-letter indicator	See Appendix	125
⠨⠐⠅	less than sign with curved sides	≺	76
⠨⠐⠅⠈⠨⠐⠅	nest of two less than signs with curved sides	⪻	82
⠨⠨	second inner radical indicator		54
	lower-case italic Greek-letter indicator	See Appendix	125
⠨⠨⠷	left angular parenthesis	⟨	94
⠨⠨⠾	right angular parenthesis	⟩	94
⠨⠨⠂	greater than sign with curved sides	≻	75
⠨⠨⠂⠈⠨⠨⠂	nest of two greater than signs with curved sides	⪼	82
⠨⠨⠨	third inner radical indicator		54
⠨⠨⠠	capital italic Greek-letter indicator	See Appendix	125
⠨⠰	lower-case italic Roman-letter indicator	See Appendix	125
⠨⠰⠠	capital italic Roman-letter indicator	See Appendix	125
⠨⠠	capital Greek-letter indicator	See Appendix	125
⠨⠠⠷	left enlarged brace	{	56

61	⠨ (Cont.)			
	⠨⠠⠾	right enlarged brace	}	56
62	⠰ (dots 5-6)			
	⠰	depresses nearer arrowhead by 45 degrees		87
		Roman-letter indicator		See Appendix 125, 26
		subscript indicator		34
	⠰⠣	left-pointing caret	<	45
	⠰⠩	right-pointing caret	>	45
	⠰⠆	proportion sign (as)	::	76 ✓
	⠰⠶	left literary brace	{	9
	⠰⠘	superscript to subscript indicator		34
	⠰⠘⠘	superscript to superscript to subscript indicator		35
	⠰⠘⠰	subscript to superscript to subscript indicator		35
	⠰⠰	subscript to subscript indicator		34
	⠰⠰⠘	superscript to subscript to subscript indicator		35
	⠰⠰⠰	subscript to subscript to subscript indicator		35
	⠰⠠	capital Roman-letter indicator		See Appendix 125
63	⠠ (dot 6)			
	⠠	capitalization indicator		See Appendix 125, 26
		enlargement indicator		56
		mathematical comma	,	9 ✓
	⠠⠕	barbed right lower arrowhead		88
	⠠⠽	curved right lower arrowhead		89

63 ⠠ (Cont.)

			Page
⠠⠯	ampersand	&	69
	curved left lower arrowhead	◟	89
⠠⠿	blunted left lower arrowhead	⌊	88
	blunted right lower arrowhead	⌋	89
	infinity	∞	98
⠠⠷	left enlarged parenthesis	(	57
⠠⠾	right enlarged parenthesis	)	57
⠠⠡	therefore	∴	99 ✓
⠠⠣	reverse pipestrap	‿	76
⠠⠹	opening complex-fraction indicator		15
⠠⠳	single enlarged vertical bar	|	57
	straight left lower arrowhead	⌈	89
	straight right lower arrowhead	⌉	89
⠠⠳⠠⠳	double enlarged vertical bar	‖	57
⠠⠳ ⠠⠳ ⠒⠒⠒⠒	normal synthetic division format	└──	108
⠠⠳ ⠠⠳ ⠒⠒⠒⠒	reverse synthetic division format	──┘	108

63 ⠠ (Cont.)

			Page
⠠⠪	barbed left lower arrowhead	↖	88
⠠⠂	comma as operation sign	,	69
⠠⠶	left literary bracket	[	9
⠠⠦	left inner quotation mark	‘	10
⠠⠌	complex fraction line	——	16, 70
⠠⠼	closing complex-fraction indicator		15
⠠⠄	ditto mark	"	9 ✓
⠠⠠	Hebrew-letter indicator	See Appendix	125, 26
⠠⠠⠹	opening hypercomplex-fraction indicator		15
⠠⠠⠼	closing hypercomplex-fraction indicator		15
⠠⠠⠠	mathematical ellipsis	...	9

GENERAL INDEX

In the GENERAL INDEX which follows, the principal entries are distinguished from the subentries by being printed in capitalized letters, the purpose being to provide a means of rapid identification.

In reading a subentry, the dash therein is to be replaced by the word or phrase of the principal entry where it occurs. If the dash is followed by an s, the combination is to be replaced by the plural of the word or phrase of the principal entry. In alphabetizing the subentries, the dash is taken to be the alphabetic equivalent of the principal entry in which it occurs.

The word *section* which heads the first reference column is generic and refers to rules as well as to subsections. Roman numerals in this column refer to rule numbers.

The page numbers in the second reference column refer to pages on which rules, sections, or subsections begin. Therefore, the entire listing at the beginning of a rule, or the entire section or subsection should be consulted, particularly if they run over to another page. An exception to this procedure has been made only in the case of RULE XVI where references are to pages 75, 76, 77, 80, or 81, according as the corresponding entry refers to a comparison sign which is, respectively, simple, modified, compounded vertically, compounded horizontally, or compounded by superposition.

Section Page
ABBREVIATED SCIENTIFIC WORD OR PHRASE XX 103
166 106
— at superscript or subscript level 73e 42
capitalization of initial letter in — 166b 106
consecutive —s 180a 117
contractions in — 108d 68
164a 104
166a 106
numeric indicator with — 4a 3
other —s 166 106
spacing with — 180 117
ABBREVIATIONS XX 103
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE, ABBREVIATIONS OF MEASUREMENT, LITERARY ABBREVIATIONS)
ABBREVIATIONS OF MEASUREMENT 164 104
— in fractions 39c 21
contractions in — 108e 68
164a 104
period after — 27 14
164a 104
single Roman letter before — 51d 30
spacing with — 179 117
(see also WHOLE WORD MEASUREMENT)
ABSOLUTE VALUE 149b 97
ACUTE ANGLE XIII 58
ACUTE TRIANGLE XIII 58
ADDITION
separation line in — 171a 110
ADJACENT ANGLES XIII 58
ALIGNMENT
— of fractions 186 123
spacing to achieve — 175d 114
ALPHABET VII 26
48 27
Cyrilic — APPENDIX 129
German — APPENDIX 126
Greek — APPENDIX 127
Hebrew — APPENDIX 128
other —s 48b 27
Roman — APPENDIX 128
Russian — APPENDIX 129
ALPHABETIC INDICATOR VII 26
49 27
54 31
capitalization indicator with — 49 27
effectiveness of — 50 28
type-form indicator with — 54 31
(see also CYRILIC-LETTER INDICATOR, GERMAN-LETTER INDICATOR, GREEK-LETTER INDICATOR, HEBREW-LETTER INDICATOR, ROMAN-LETTER INDICATOR, RUSSIAN-LETTER INDICATOR)
ALTERNATE EXTERIOR ANGLES XIII 58
ALTERNATE INTERIOR ANGLES XIII 58
AMPERSAND XV 69
109 71
AMPLITUDE XX 103
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE)
AND XV 69
ANGLE XIII 58
101b 63
acute — XIII 58
adjacent —s XIII 58
alternate exterior —s XIII 58
alternate interior —s XIII 58
— with interior arc XIII 58
— with superposed arc XIII 58
complementary —s XIII 58
corresponding —s XIII 58
exterior —s XIII 58
interior —s XIII 58
obtuse — XIII 58
right — XIII 58
straight — XIII 58
supplementary —s XIII 58
vertical —s XIII 58
(see also ANGLE WITH INTERIOR ARROW)
ANGLE BRACKET XVIII 94
144 95
left — XVIII 94
right — XVIII 94
(see also DISTINCTION BETWEEN)
ANGLE WITH INTERIOR ARROW
angle with interior clockwise arrow XIII 58
angle with interior counterclockwise arrow XIII 58
ANGSTROM UNIT XIX 98
ANGULAR PARENTHESES XVIII 94
(see also ANGLE BRACKET)
ANTILOGARITHM XX 103
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE)
APOSTROPHE III 8
(see also APOSTROPHE-S)
APOSTROPHES 15 11
— with letters 15 11
— with mathematical expressions 15 11
— with numerals 15 11
punctuation of the — combination 15 11
Section Page
APPROACHES 131a 83
ARC XX 103
— opening downward IX 44
86a 48
XIII 58
XVI 75
131a 83
— opening upward IX 44
86b 49
XIII 58
XVI 75
131b 83
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE, ANGLE)
ARGUMENT XX 103
criteria for terminating — 181 118
definition of — 181 118
numeric indicator with — 181 118
spacing rule for — 182 119
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE)
ARROW XVI 75
131c 83
— as modifier 87 49
— with vectors 58b 33
equilateral triangle under — 92 52
left parenthesis on counterclockwise — shaft XVI 81
left-pointing — XVI 75
representation of —s XVII 87
right-pointing — XVI 75
131c 83
two-way — XVI 75
131c 83
(see also ANGLE WITH INTERIOR ARROW, ARROW COMBINATIONS, ARROW COMPONENTS, ARROW DIRECTIONS, ARROW EAST, ARROW NORTH, ARROW SHAFT, ARROW SOUTH, ARROW TYPE, ARROW WEST, ARROWHEAD, BARBED AND DOTTED ARROWS, BOLDFACE ARROW, CIRCLE WITH INTERIOR ARROWS, SIX-STEP RULE FOR CONSTRUCTION OF ARROWS, TRANSCRIBER'S NOTE CONCERNING)
ARROW COMBINATIONS
— compounded horizontally XVI 80
134 87
— compounded vertically XVI 77
133a 86
ARROW COMPONENTS XVII 87
136 89
ARROW DIRECTIONS XVII 87
138 90

Section Page
— in six-step rule for construction
of arrows 137b 89
directly-over indicator with — 138c 90
directly-under indicator with — 138b 90
horizontal — 138a 90
oblique — 138c 90
subscript indicator with — 138c 90
superscript indicator with — 138c 90
vertical — 138b 90
ARROW EAST
— over arrow west XVI 77
— over boldface arrow west XVI 77
— with upper barb over arrow
west with lower barb XVI 77
long — over short arrow west XVI 77
short — over long arrow west XVI 77
vertical bar through shaft of — XVI 81
(see also ARROW WEST, BOLDFACE ARROW, CIRCLE WITH INTERIOR ARROWS)
ARROW NORTH
— followed by arrow south XVI 80
— followed by boldface arrow
south XVI 80
(see also ARROW SOUTH, BOLDFACE ARROW, CIRCLE WITH INTERIOR ARROWS)
ARROW SHAFT 139 91
— in six-step rule for construction
of arrows 137e 89
left parenthesis on counterclockwise — XVI 81
length of — 139c 91
(see also CURVED ARROW SHAFT, DOUBLE ARROW SHAFT, LONG ARROW SHAFT, ORDINARY ARROW SHAFT, SHORT ARROW SHAFT)
ARROW SOUTH
— followed by arrow north XVI 80
— followed by boldface arrow
north XVI 80
(see also ARROW NORTH, BOLDFACE ARROW, CIRCLE WITH INTERIOR ARROWS)
ARROW TYPE
— in six-step rule for construction
of arrows 137c 89
boldface — XVII 87
140 92
ordinary — XVII 87
140 92
ARROW WEST
— over boldface arrow east XVI 77
vertical bar through shaft of — XVI 81
(see also ARROW EAST, BOLDFACE ARROW, CIRCLE WITH INTERIOR ARROWS)
ARROWHEAD 141 92
— combinations 141b 93
depresses nearer — by 45
degrees XVII 87
138c 90
elevates nearer — by 45 degrees XVII 87
138c 90
makes nearer — point north XVII 87
138b 90
makes nearer — point south XVII 87
138b 90
(see also BARBED ARROWHEAD, BLUNTED ARROWHEAD, CURVED ARROWHEAD, FULL ARROWHEAD, LEFT ARROWHEAD, LOWER ARROWHEAD, RIGHT ARROWHEAD, STRAIGHT ARROWHEAD, UPPER ARROWHEAD)
AS XVI 75
131p 86

Section Page
ASTERISK III 8
16 12
XV 69
110 71
— in references 16 12
110 71
— in superscript position 16 12
AT XIX 98
150 99
BAR
— over double tilde XVI 77
— over greater than XVI 77
— over inclusion XVI 77
— over integral sign 159 101
— over less than XVI 77
— over logical product XVI 77
— over logical sum XVI 77
— over reverse inclusion XVI 77
— over single tilde XVI 77
133c 87
— under double tilde XVI 77
— under greater than XVI 77
— under inclusion XVI 77
— under integral sign 159 101
— under intersection XVI 77
— under less than XVI 77
— under logical product XVI 77
— under logical sum XVI 77
— under reverse inclusion XVI 77
— under single tilde XVI 77
— under union XVI 77
double — 81 47
(see also BAR OVER AND BAR UNDER, BAR OVER AND EQUALS SIGN UNDER, EQUALS SIGN OVER AND BAR UNDER, HORIZONTAL BAR, VERTICAL BAR)
BAR OVER AND BAR UNDER
— logical product XVI 77
— logical sum XVI 77
BAR OVER AND EQUALS SIGN UNDER
— logical product XVI 77
133e 87
— logical sum XVI 77
133e 87
BARBED AND DOTTED ARROWS IX 44
barbed at both ends IX 44
87a 49
barbed at left IX 44
87b 49
barbed at left, dotted at right IX 44
87c 49
barbed at right IX 44
87d 49
dotted at both ends IX 44
87e 49
dotted at left, barbed at right IX 44
87g 50
dotted at left, no barb IX 44
87f 50
dotted at right, no barb IX 44
87h 50
BARBED ARROWHEAD
left full — XVII 87
left lower — XVII 87
left upper — XVII 87
right full — XVII 87
right lower — XVII 87
right upper — XVII 87
BARRED BRACE XVIII 94
left — XVIII 94
right — XVIII 94
BARRED BRACKET XVIII 94
left — XVIII 94
right — XVIII 94
BASE LINE
definition of — 63 35
BASE-LINE INDICATOR VIII 34
non-use of — 72 41
78e 46
use of — 71 40
(see also DISTINCTION BETWEEN)

Section Page
BASIC SHAPES XIII 58
BECAUSE XIX 98
BELONGS TO 131l 85
BINOMIAL COEFFICIENT 82 47
directly-under indicator in — 82 47
BLUNTED ARROWHEAD
left full — XVII 87
left lower — XVII 87
left upper — XVII 87
right full — XVII 87
right lower — XVII 87
right upper — XVII 87
BOLDFACE ARROW
— east over arrow west XVI 77
— east over boldface arrow west XVI 77
— north followed by arrow south XVI 80
— north followed by boldface
arrow south XVI 80
— south followed by arrow north XVI 80
— south followed by boldface
arrow north XVI 80
— type XVII 87
140 92
— west over arrow east XVI 77
— west over boldface arrow east XVI 77
(see also ARROW EAST, ARROW NORTH, ARROW SOUTH, ARROW WEST)
BOLDFACE BRACKET XVIII 94
146 96
left — XVIII 94
right — XVIII 94
BOLDFACE COMPARISON SIGNS 58a 32
BOLDFACE EQUALS SIGN XVI 75
BOLDFACE TYPE 58 32
— with vectors 58b 33
(see also BOLDFACE ARROW, BOLDFACE BRACKET, BOLDFACE COMPARISON SIGNS, BOLDFACE EQUALS SIGN, BOLDFACE-TYPE INDICATOR, BOLDFACE VERTICAL BAR)
BOLDFACE-TYPE INDICATOR VII 26
— with comparison symbol 58a 32
— with filled-in shapes 101a 63
— with operation symbol 58a 32
— with shape symbol 58a 32
101a 63
(see also MATHEMATICAL SIGNIFICANCE)
BOLDFACE VERTICAL BAR XVIII 94
double — XVIII 94
single — XVIII 94
BORROWING 44b 25
(see also CARRYING AND BORROWING)
BRACE
left — III 8
XVIII 94
literary — 24 13
mathematical — 147 96
right — III 8
XVIII 94
(see also BARRED BRACE, ENLARGED BRACE, HORIZONTAL GROUPING SIGNS, USE OF LITERARY GROUPING SYMBOLS, USE OF MATHEMATICAL GROUPING SYMBOLS)
BRACKET
left — III 8
XVIII 94
literary — 24 13
mathematical — 147 96
right — III 8
XVIII 94
(see also ANGLE BRACKET, BARRED BRACKET, BOLDFACE BRACKET, ENLARGED BRACKET, HORIZONTAL GROUPING SIGNS, USE OF LITERARY GROUPING SYMBOLS, USE OF MATHEMATICAL GROUPING SYMBOLS)

	Section	Page
BRAILLE INDICATOR	I	1
— with shape modification symbol	178a	116
definition of —	I	1
CANCELLATION	VI	24
— in computation	44	24
linear arrangement within spatial arrangement with —	44a	24
(see also CANCELLATION INDICATOR, CANCELLATION STROKE)		
CANCELLATION INDICATOR	VI	24
— with level indicator	73c	42
closing —	VI	24
opening —	VI	24
use of —	44	24
CANCELLATION STROKE		
direction of —	44a	24
(see also DISTINCTION BETWEEN)		
CAP		
(see INTERSECTION)		
CAPITALIZATION	VII	26
—of initial letter in abbreviated scientific word or phrase	166b	106
— of letters	45a	26
	46	26
	47a	27
	47c	27
	48b	27
— of Roman numerals	10	8
	45b	26
CAPITALIZATION INDICATOR	VII	26
— with alphabetic indicator	49	27
— with shape indicator	48b	27
contractions for to, into, by before —	108a	67
effectiveness of —	47	27
non-use of —	46	26
use of —	45	26
CAPITALIZED LETTER		
(see CAPITALIZATION, CIRCLE WITH INTERIOR)		
CARET	IX	44
	90a	51
	XIX	98
	151	99
— modifying horizontal bar	88b	50
	XVI	76
	131g	84
— modifying tilde	95	53
— over equals sign	XVI	76
	132	86
— under equals sign	XVI	76
	132	86
(see also INVERTED CARET, LEFT-POINTING CARET, RIGHT-POINTING CARET)		
CARRYING AND BORROWING		
separation line in —	XXI	107
	171b	111
CARTESIAN PRODUCT	125a	74
CENT SIGN	XIX	98
	152	99
CHAIN OF MATHEMATICAL EXPRESSIONS	184d	122
CHANGING NUMBER FORMS		
separation line in —	XXI	107
CHARTS		
(see TABLES AND CHARTS)		
CHECK MARK	III	8
	17	12
spacing with —	17	12
CIRCLE	XIII	58
	101c	63
integral with superposed —	XIX	98
small —	XIII	58
(see also CIRCLE WITH INTERIOR, CIRCLE WITH INTERIOR ARROWS, CIRCLE WITH SUPERPOSED)		
CIRCLE WITH INTERIOR		
— capitalized letter	XIII	58
— cross	XIII	58
— dot	XIII	58
	XV	69
	111	71
— horizontal bar between two vertical bars	XIII	58
— minus sign	XIII	58
— plus sign	XIII	58
	XV	69
	112	71
(see also CIRCLE WITH INTERIOR ARROWS)		
CIRCLE WITH INTERIOR ARROWS		
circle with interior arrow east	XIII	58
circle with interior arrow east over interior arrow west	XIII	58
circle with interior arrow north	XIII	58
circle with interior arrow north followed by interior arrow south	XIII	58
circle with interior arrow south	XIII	58
circle with interior arrow south followed by interior arrow north	XIII	58
circle with interior arrow west	XIII	58
circle with interior arrow west over interior arrow east	XIII	58
CIRCLE WITH SUPERPOSED		
— horizontal bar	XIII	58
— vertical bar	XIII	58
CIRCUMFLEX		
(see CARET)		
CLOCKWISE ARROW	139a	91
(see also ANGLE WITH INTERIOR ARROW, CURVED ARROW SHAFT)		
CLOSED INTERVAL	87e	49
CLOSING		
(see CANCELLATION INDICATOR, COMPLEX-FRACTION INDICATOR, FRACTIONAL-PART-OF-A-MIXED-NUMBER INDICATOR, HYPERCOMPLEX-FRACTION INDICATOR, SIMPLE-FRACTION INDICATOR)		
COLOGARITHM	XX	103
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE)		
COLON	III	8
COMES FROM	131c	83
COMMA	III	8
	18	12
	XV	69
	113	71
— as sign of operation	113	71
— in long numerals	6	7
— in special arrangement for multiplication	174	113
— in subscripts	73b	41
level indicator with —	73b	41
literary —	18	12
mathematical —	18	12
punctuation indicator with —	13a	10
spacing with mathematical —	18	12
(see also DISTINCTION BETWEEN, MATHEMATICAL COMMA)		
COMPARISON SIGNS	XVI	75
— compounded by superposition	XVI	81
	135	87
— compounded horizontally	XVI	80
	134	87
— compounded vertically	XVI	77
	131g	84
	131r	86
	133	86
modified —	XVI	76
	131g	84
	131r	86
	132	86
simple —	XVI	75
	131	83
COMPARISON SYMBOLS		
multipurpose indicator with —	96c	54
punctuation of —	178c	117
spacing with —	176	115
COMPLEMENTARY ANGLES	XIII	58
COMPLEX FRACTION	33	18
definition of —	33a	18
simple-fraction indicator with —	30c	17
(see also COMPLEX-FRACTION INDICATOR, COMPLEX FRACTION LINE)		
COMPLEX-FRACTION INDICATOR	IV	15
closing —	IV	15
non-use of —s	35	18
opening —	IV	15
use of —s	34	18
COMPLEX FRACTION LINE	IV	15
	XV	69
COMPUTATION		
cancellation in —	44	24
numeric indicator in —	4d	4
Roman letters in —	52c	30
Roman numerals in —	10b	8
CONSTRUCTION OF ARROWS		
(see SIX-STEP RULE FOR CONSTRUCTION OF ARROWS)		
CONTAINS	XVI	75
	131j	85
— the element	XVI	75
CONTINUED FRACTION	38	20
definition of —	38	20
fraction indicators with —	39b	21
spatial arrangement in —	39b	21
CONTRACTABLE COMBINATIONS		
(see CONTRACTIONS)		
CONTRACTIONS	XIV	67
— in abbreviated scientific word or phrase	108d	68
	166a	106
— in abbreviations of measurement	108e	68
	164a	104
— in name of letter	108c	68
— in name of shape	48b	27
	101	63
	108c	68
— in name of shape modifier	102	64
	108c	68
—in ordinal endings	7	7
	108b	68
— in word or phrase in the role of a mathematical expression	108f	68
	177	116
— likely to be mistaken for mathematical symbol	108g	69
non-use of —	108	67
use of — and short form words	107	67
(see also CONTRACTIONS FOR TO, INTO, BY)		
CONTRACTIONS FOR TO, INTO, BY	108a	67
— before capitalization indicator	108a	67
— before italic-type indicator	108a	67
— before mathematical expression	108a	67
— before numeric indicator	108a	67
— before Roman-letter indicator	108a	67
CORNER	XIII	58
	105	66
—s in division format	105a	66
drawing of —s	105b	66
northeast —	XIII	58
	105c	66
northwest —	XIII	58
representation of —s	105a	66
southeast —	XIII	58
southwest —	XIII	58
	105d	67
termination indicator with —s	105a	66

	Section	Page
CORRESPONDENCE		
perspective —	XVI	76
projective —	XVI	76
	132	86
CORRESPONDING ANGLES	XIII	58
CORRESPONDS TO	131c	83
COSECANT	XX	103
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE, HYPERBOLIC)		
COSINE	XX	103
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE, HYPERBOLIC)		
COTANGENT	XX	103
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE, HYPERBOLIC)		
COUNTERCLOCKWISE ARROW	139a	91
left parenthesis on — shaft	XVI	81
(see also ANGLE WITH INTERIOR ARROW, CURVED ARROW SHAFT)		
COVERSINE	XX	103
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE)		
CRITERIA FOR TERMINATING ARGUMENT	181	118
CROSS	XV	69
	125a	74
(see also CIRCLE WITH INTERIOR)		
CROSSED D	XIX	98
CROSSED H	XIX	98
	153	100
CROSSHATCH		
(see NUMBER SIGN)		
CUP		
(see UNION)		
CURLY BRACKET		
(see BRACE)		
CURVED ARROW SHAFT	XVII	87
	139	91
CURVED ARROWHEAD		
left full —	XVII	87
left lower —	XVII	87
left upper —	XVII	87
right full —	XVII	87
right lower —	XVII	87
right upper —	XVII	87
CURVED SIDES		
(see DIVISION FORMAT, NEST OF)		
CYRILIC ALPHABET	48a	27
	APPENDIX	129
CYRILIC-LETTER INDICATOR	VII	26
DAGGER	III	8
	16	12
— in references	16	12
— in superscript position	16	12
DASH	III	8
numeric indicator with —	3	1
	4	3
punctuation indicator with —	13a	10
(see also LONG DASH, SHORT DASH)		
DECIMAL		
(see DECIMAL POINT, RECURRING DECIMAL)		
DECIMAL POINT	XIX	98
	154	100
— in special arrangement for multiplication	174	113
multipurpose indicator with —	96d	54
numeric indicator with —	3	1
	4	3
DEFINITION OF		
— argument	181	118
— base line	63	35
— braille indicator	1	1
— chain of mathematical expressions	184d	122
— complex fraction	33a	18
— continued fraction	38	20
— direct subscript	69	39
— direct superscript	69	39
— hypercomplex fraction	36a	19
— linked mathematical expressions	184d	122
— mixed number	32a	18
— modifier	77	45
— radicand	97d	55
— shape signs	100	62
— shape modified by superposition	104	65
— shape with interior modification	103a	65
— shape with structural modification	102	64
— simple fraction	29	16
— subscript	63	35
— superscript	63	35
— word or phrase in the role of a mathematical expression	148b	97
DEGREE SIGN	XV	69
	118	72
— over equals sign	XVI	76
	132	86
DEL	XIX	98
DEPRESSES NEARER ARROWHEAD BY 45 DEGREES	XVII	87
	138c	90
DETERMINANT		
— as system of mathematical expressions	98	57
numeric indicator with —s	4b	3
DIAGONAL		
(see DIAGONAL FRACTION LINE, DIAGONAL LINE, DIAGONAL NEGATION STROKE, SQUARE WITH INTERIOR)		
DIAGONAL FRACTION LINE	IV	15
	XV	69
DIAGONAL LINE	IV	15
— in mixed numbers	32a	18
— in simple fractions	30b	16
	31b	17
(see also DIAGONAL NEGATION STROKE)		
DIAGONAL NEGATION STROKE	44b	25
direction of —	130	82
(see also DISTINCTION BETWEEN)		
DIAGRAMS		
(see FIGURES AND DIAGRAMS)		
DIAMOND	XIII	58
	101d	64
DIFFERENCE BETWEEN	XVI	77
DIRECT SUBSCRIPTS AND SUPERSCRIPTS	69	39
definition of —	69	39
DIRECTION		
(see ARROW DIRECTIONS, CANCELLATION STROKE, DIAGONAL NEGATION STROKE)		
DIRECTLY-OVER INDICATOR	IX	44
— in arrow directions	138b	90
— in five-step rule for transcribing modified expressions	78c	46
DIRECTLY-UNDER INDICATOR	IX	44
— in arrow directions	138b	90
— in binomial coefficient	82	47
— in five-step rule for transcribing modified expressions	78c	46
DISTINCTION BETWEEN		
— base-line indicator and multipurpose indicator	96	53
— cancellation stroke and diagonal negation stroke	44b	25
	130	82
— comma as punctuation and comma as sign of operation	113	71
— double bar and equals sign	88d	51
— empty set and phi	155	100
— exclamation point and factorial sign	21	13
— greater than and right angle bracket	144	95
— hyphen and minus sign	22	13
— identity sign and triple bar	88d	51
— interior modification and superposition	104	65
— left angle bracket and less than	144	95
— lower-case epsilon and the membership sign	131l	85
DITTO MARKS	III	8
	17	12
spacing with —	17	12
DIVIDED BY	XV	69
	114	72
DIVISION	XV	69
	114	72
separation line in —	171c	111
(see also DIVISION FORMAT, SYNTHETIC DIVISION FORMAT)		
DIVISION FORMAT	170	108
inverted — with curved side	XXI	107
inverted — with straight side	XXI	107
normal — with curved side	XXI	107
normal — with straight side	XXI	107
remainder in —	170b	108
separation line in —	170a	108
DOLLAR SIGN	XIX	98
	152	99
DOT	IX	44
	91	52
— between bars of equals sign	XVI	81
— modifying horizontal bar	88b	50
	XVI	76
	131g	84
— modifying tilde	95	53
— over equals sign	XVI	76
	132	86
— under simple tilde	XVI	76
— with recurring decimal	91a	52
— within inclusion	XVI	81
— within reverse inclusion	XVI	81
filled-in —	XV	69
	115	72
hollow —	IX	44
	93	53
	XV	69
	118	72
(see also CIRCLE WITH INTERIOR, DOT OVER AND DOT UNDER, SQUARE WITH INTERIOR)		
DOT OVER AND DOT UNDER		
— equals sign	XVI	76
	132	86
two —s equals sign	XVI	76
	132	86
DOUBLE ARROW SHAFT		
long —	XVII	87
ordinary —	XVII	87
short —	XVII	87
DOUBLE BAR	81	47
(see also DISTINCTION BETWEEN)		
DOUBLE CAPITAL		
— in Roman numerals	10a	8
	47b	27
DOUBLE INTEGRAL	XIX	98
DOUBLE TILDE	XVI	77
	133d	87
(see also BAR, EQUALS SIGN)		
DOUBLE VERTICAL BAR	XVIII	94
(see also BOLDFACE VERTICAL BAR, ENLARGED VERTICAL BAR)		
DRAWING OF		
— corners	105b	66
— enlarged grouping symbols	98	57
— horizontal grouping symbols	145	95
— letters from other alphabets	48b	27
— shapes	100	62
EAST		
(see ARROW EAST, ARROW WEST, CIRCLE WITH INTERIOR ARROWS)		

Section Page
EFFECTIVENESS OF
— alphabetic indicator 50 28
— capitalization indicator 47 27
— level indicator 73 41
— numeric indicator 5 7
— punctuation indicator 14 11
— type-form indicator 56 32
ELEVATES NEARER ARROWHEAD BY 45 DEGREES XVII 87
138c 90
ELLIPSE XIII 58
101e 64
ELLIPSIS III 8
20 13
— as omission sign 41 23
literary — 20 13
mathematical — 20 13
numeric indicator after — 4c 4
punctuation of mathematical — 20 13
EMPTY SET XIX 98
155 100
(see also DISTINCTION BETWEEN)
ENCLOSED LETTERS
single — 26a 14
ENCLOSED NUMERALS
numeric indicator with — 148a 96
single — 26a 14
END OF PROOF 149c 97
ENGLISH BRAILLE
— numerals 2 1
— page numbers 2 1
— Roman numerals 10a 8
10d 8
— volume numbers 2 1
ENLARGED BRACE
left — XII 56
right — XII 56
ENLARGED BRACKET
left — XII 56
right — XII 56
ENLARGED GROUPING SIGNS XII 56
143 95
— with fractions 99b 58
(see also ENLARGED GROUPING SYMBOLS)
ENLARGED GROUPING SYMBOLS 177 115
drawing of — 98 57
(see also ENLARGED GROUPING SIGNS)
ENLARGED PARENTHESIS
left — XII 56
right — XII 56
ENLARGED VERTICAL BAR XII 56
double — XII 56
single — XII 56
ENLARGEMENT INDICATOR XII 56
non-use of — 99 58
use of — 98 57
EQUALS SIGN 81 47
XVI 75
131d 84
boldface — XVI 75
dot between bars of — XVI 81
— over double tilde XVI 77
— over greater than XVI 77
— over inclusion XVI 77
— over less than XVI 77
— over logical product XVI 77
133e 87
— over logical sum XVI 77
133e 87
— over reverse inclusion XVI 77
— over single tilde XVI 77
— through inclusion XVI 81
— through reverse inclusion XVI 81
— under double tilde XVI 77
— under greater than XVI 77
— under inclusion XVI 77
— under intersection XVI 77
— under less than XVI 77
— under logical product XVI 77
133e 87

Section Page
— under logical sum XVI 77
133e 87
— under reverse inclusion XVI 77
— under single tilde XVI 77
— under union XVI 77
133b 86
horizontal bar instead of — 133b 84
(see also BAR OVER AND EQUALS SIGN, CARET, DEGREE SIGN, DISTINCTION BETWEEN, DOT, DOT OVER AND DOT UNDER, EQUALS SIGN OVER AND BAR UNDER, EQUALS SIGN OVER AND EQUALS SIGN UNDER, EQUILATERAL TRIANGLE, GREATER THAN, HOLLOW DOT, INVERTED CARET, LEFT-POINTING CARET, LESS THAN, QUESTION MARK, RIGHT-POINTING CARET, VERTICAL BAR)
EQUALS SIGN OVER AND BAR UNDER
— logical product XVI 77
133e 87
— logical sum XVI 77
133e 87
EQUALS SIGN OVER AND EQUALS SIGN UNDER
— logical product XVI 77
133e 87
— logical sum XVI 77
133e 87
EQUILATERAL TRIANGLE IX 44
92 52
XIII 58
— over equals sign XVI 76
— under arrow 92 52
EQUIVALENCE XVI 77
ERROR FUNCTION XX 103
(see also ABBREVIATED SCIENTRIFIC WORD OR PHRASE)
EXCESS XVI 75
EXCLAMATION POINT III 8
21 13
(see also DISTINCTION BETWEEN)
EXERCISES
runover in — 184c 121
subdivisions in — 184c 121
EXISTENTIAL QUALIFIER XIX 98
156 100
EXPONENTIAL XX 103
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE)
EXSECANT XX 103
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE)
EXTENDED TILDE IX 44
95 53
XVI 75
131s 86
EXTERIOR ANGLES XIII 58
EXTRACTION OF A ROOT
separation line in — 171c 111
FACTORIAL XIII 58
105d 67
XIX 98
157 100
(see also DISTINCTION BETWEEN)
FEET 160 101
164c 105
FIGURES AND DIAGRAMS
numerals in — 9 8
FILLED-IN DOT XV 69
115 72
FILLED-IN SHAPE 101a 63
(see also BOLDFACE-TYPE INDICATOR, REPRESENTATION OF)
FILLED-IN SQUARE XV 69
116 72

Section Page
FIVE-STEP RULE FOR TRANSCRIBING MODIFIED EXPRESSIONS 78 45
modification indicator in — 78c 46
modified expression in — 78b 46
modifier in — 78d 46
multipurpose indicator in — 78a 45
termination indicator in — 78e 46
(see also DIRECTLY-OVER INDICATOR, DIRECTLY-UNDER INDICATOR)
FOR ALL XIX 98
FOR EACH XIX 98
FOR EVERY XIX 98
FOR SOME XIX 98
FORMAT SIGNS XXI 107
FORMAT SYMBOLS XXI 107
FORMULAS
Roman letters in — 52c 30
Roman numerals in — 10b 8
(see also REFERENCES)
FRACTION IV 15
allignment of —s 186 123
complex — 33 18
continued — 38 20
hypercomplex — 36 19
linear arrangement within spatial arrangement with —s 39a 20
numeric indicator with —s 4d 4
simple — 29 16
(see also ABBREVIATIONS OF MEASUREMENT, COMPLEX FRACTION, CONTINUED FRACTION, ENLARGED GROUPING SIGNS, FRACTION INDICATOR, FRACTION LINE, HYPERCOMPLEX FRACTION, SPATIAL ARRANGEMENT, WORD OR PHRASE IN THE ROLE OF A MATHEMATICAL EXPRESSION)
FRACTIONAL INDICATOR IV 15
— in continued fractions 39b 21
(see also COMPLEX FRACTION INDICATOR, HYPERCOMPLEX FRACTION INDICATOR, SIMPLE FRACTION INDICATOR)
FRACTION LINE IV 15
XV 69
complex — IV 15
XV 69
diagonal — IV 15
XV 69
horizontal —s IV 15
simple — IV 15
XV 69
FRACTIONAL PART OF A MIXED NUMBER INDICATOR IV 15
FULL ARROWHEAD
barbed left — XVII 87
barbed right — XVII 87
blunted left — XVII 87
blunted right — XVII 87
curved left — XVII 87
curved right — XVII 87
straight left — XVII 87
straight right — XVII 87
FULL LINE 87a 49
FUNCTION
(see HYPERBOLIC FUNCTION, INTEGER FUNCTION, TRIGONOMETRIC FUNCTION)
GERMAN ALPHABET 48a 27
APPENDIX 126
GERMAN LETTER INDICATOR XII 26
GREATER THAN XVI 75
131f 84
— followed by equals sign followed by less than XVI 80
— followed by less than XVI 80
— with curved sides XVI 75
(see also BAR, DISTINCTION BETWEEN, EQUALS SIGN, LESS THAN, NEST OF)

	Section	Page
GREEK ALPHABET	48a	27
	APPENDIX	127
GREEK LETTER INDICATOR	VII	26
GROUPING SIGNS	XVIII	94
	142	95
(see also ENLARGED GROUPING SIGNS, HORIZONTAL GROUPING SIGNS)		
GROUPING SYMBOLS		
numerals within —	4b	3
	148a	96
Roman numerals within —	148a	96
spacing with —	177	115
(see also ENLARGED GROUPING SYMBOLS, MATHEMATICAL GROUPING SYMBOLS, WORD OR PHRASE IN THE ROLE OF A MATHEMATICAL EXPRESSION)		
HALF-LINE		
— extending to the left	87b	49
— extending to the right	87d	49
HALF-OPEN INTERVAL		
— closed at the left and open at the right	87f	50
— open at the left and closed at the right	87h	50
HAVERSINE	XX	103
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE)		
HEAT		
(see IN THE PRESENCE OF HEAT)		
HEBREW ALPHABET	48a	27
	APPENDIX	128
HEBREW LETTER INDICATOR	VII	26
HEXAGON	XIII	58
irregular —	XIII	58
regular —	XIII	58
HOLLOW DOT	IX	44
	93	53
	XV	69
	118	72
— as superscript	118	72
— over equals sign	93	53
HOLLOW SQUARE	XV	69
	119	72
HORIZONTAL		
(see HORIZONTAL ARROW DIRECTIONS, HORIZONTAL BAR, HORIZONTAL FRACTION LINE, HORIZONTAL GROUPING SIGNS)		
HORIZONTAL ARROW DIRECTIONS	138a	90
HORIZONTAL BAR	IX	44
	81	47
	88	50
	XVI	75
	131g	84
— instead of equals sign	131g	84
— over simple comparison sign	131g	84
— through inclusion	XVI	81
— through reverse inclusion	XVI	81
— under simple comparison sign	131g	84
— with integral sign	88e	51
— with limit	88e	51
— with recurring decimal	88c	50
(see also CARET, CIRCLE WITH INTERIOR, CIRCLE WITH SUPERPOSED, DOT, SQUARE WITH INTERIOR, SQUARE WITH SUPERPOSED)		
HORIZONTAL FRACTION LINE	IV	15
— for complex fractions	IV	15
— for simple fractions	IV	15
— in mixed numbers	31a	17
	32a	18
— in spatial arrangement	IV	15
HORIZONTAL GROUPING SIGNS	145	95
(see also DRAWING OF)		
HYPERBOLIC		
— cosecant	XX	103
— cosine	XX	103
— cotangent	XX	103
— secant	XX	103
— sine	XX	103
— tangent	XX	103
(see also HYPERBOLIC FUNCTION)		
HYPERBOLIC FUNCTION	168	106
HYPERCOMPLEX FRACTION	36	19
definition of —	36a	19
spatial arrangement in —	39a	20
(see also HYPERCOMPLEX FRACTION INDICATOR)		
HYPERCOMPLEX FRACTION INDICATOR	IV	15
closing —	IV	15
opening —	IV	15
use of —s	37	19
HYPHEN	III	8
	22	13
— as omission sign	41	23
— with long numerals	6	7
numeric indicator with —	3c	2
punctuation indicator with —	13a	10
(see also DISTINCTION BETWEEN)		
IDENTITY	XVI	75
	131h	84
(see also DISTINCTION BETWEEN, SPEAR)		
IF AND ONLY IF	131c	83
IMPLIES	131c	83
IN THE PRESENCE OF HEAT	92	52
INCHES	160	101
	164c	105
INCLUSION	XVI	75
	131i	84
(see also BAR, DOT, EQUALS SIGN, HORIZONTAL BAR, REVERSE INCLUSION)		
INDEX OF RADICAL		
— in six-step rule for transcribing radicals	97b	54
(see also INDEX-OF-RADICAL INDICATOR)		
INDEX-OF-RADICAL INDICATOR	XI	54
—in six-step rule for transcribing radicals	97b	54
INDICATOR		
braille —s	I	1
definition of braille —	I	1
(see also ALPHABETIC INDICATOR, CANCELLATION INDICATOR, CAPITALIZATION INDICATOR, ENLARGEMENT INDICATOR, FRACTION INDICATOR, FRACTIONAL PART OF A MIXED NUMBER INDICATOR, LEVEL INDICATOR, MODIFICATION INDICATOR, MULTIPURPOSE INDICATOR, NUMERIC INDICATOR, OMISSION INDICATOR, PUNCTUATION INDICATOR, RADICAL INDICATOR, SHAPE INDICATOR, SUPERPOSITION INDICATOR, TYPE FORM INDICATOR)		
INFIMUM	XX	103
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE)		
INFINITY	XIX	98
	158	101
(see also INTEGRAL WITH SUPERPOSED)		
INNER QUOTATION MARKS		
left —	III	8
right —	III	8
INTEGER FUNCTION	146	96
(see also BOLDFACE BRACKET)		
INTEGRAL	XIX	98
	159	101
double —	XIX	98
large —	99c	58
lower —	XIX	98
single —	XIX	98
triple —	XIX	98
upper —	XIX	98
(see also BAR, HORIZONTAL BAR, INTEGRAL WITH SUPERPOSED)		
INTEGRAL WITH SUPERPOSED		
— circle	XIX	98
— infinity sign	XIX	98
— rectangle	XIX	98
— square	XIX	98
INTERIOR ANGLES	XIII	58
INTERIOR ARC		
(see ANGLE)		
INTERIOR ARROW		
(see ANGLE WITH INTERIOR ARROW, CIRCLE WITH INTERIOR ARROWS)		
INTERIOR MODIFICATION		
(see DISTINCTION BETWEEN, SHAPE WITH INTERIOR MODIFICATION)		
INTERIOR MODIFIERS		
— arranged horizontally	103b	65
— arranged vertically	103c	65
— with signs of shape	85	48
multipurpose indicator with —	103b	65
shape modification indicator with —	103b	65
	103c	65
INTERLOCKING PARENTHESES	XVI	81
	135b	87
INTERSECTING LINES	XIII	58
INTERSECTION	XV	69
	120	72
	133b	87
(see also BAR, EQUALS SIGN)		
INTERVAL		
(see CLOSED INTERVAL, HALF-OPEN INTERVAL, OPEN INTERVAL)		
INVERTED CARET	IX	44
	90b	51
— over equals sign	XVI	76
	132	86
INVERTED DIVISION FORMAT		
(see DIVISION FORMAT)		
IRREGULAR HEXAGON	XIII	58
IRREGULAR PENTAGON	XIII	58
IRREGULAR POLYGON	101h	64
IS		
— a member of	131l	85
— a part of	XVI	75
— a subset of	XVI	75
	131i	84
— an element of	XVI	75
	131l	85
— approximately equal to	XVI	76
	132	86
— congruent to	XVI	75
	131h	84
— contained	XVI	75
	131i	84
— divisible by	133c	87
— equal in degrees to	93	53
	XVI	75
	132	86
— equal to	XVI	75
	131d	84
— equal to or greater than	XVI	77
— equal to or less than	XVI	77
— equivalent to	XVI	77
	131t	86
— greater than	XVI	75
— greater than or equal to	XVI	77
— identical to	131h	84
— identically equal to	131h	84
— implied by	131c	83
— large compared with	XVI	81
— less than	XVI	75
— less than or equal to	XVI	77

	Section	Page
— parallel to	XIII	58
	101f	64
— perpendicular to	XIII	58
	101f	64
— perspective to	XVI	76
— projective to	XVI	76
	132	86
— related to	XVI	75
	131s	86
	131t	86
— similar to	131t	86
— small compared with	XVI	81
— to	XVI	75
	131q	86
ISOSCELES TRIANGLE	XIII	58
IT DOES NOT FOLLOW THAT	162	102
ITALIC TYPE INDICATOR	VII	26
contractions for to, into, by before —	108a	67
(see also MATHEMATICAL SIGNIFICANCE)		
JOIN	XV	69
LARGE INTEGRAL	99c	58
LARGE RADICAL	99c	58
LEFT		
(see ANGLE BRACKET, BARBED ARROWHEAD, BARRED BRACE, BARRED BRACKET, BLUNTED ARROWHEAD, BOLDFACE BRACKET, BRACE, BRACKET, CURVED ARROWHEAD, ENLARGED BRACE, ENLARGED BRACKET, ENLARGED PARENTHESIS, FULL ARROWHEAD, INNER QUOTATION MARKS, LEFT ARROWHEAD, LEFT-POINTING ARROW, LEFT-POINTING CARET, LEFT PARENTHESIS ON COUNTERCLOCKWISE ARROW SHAFT, LOWER ARROWHEAD, OUTER QUOTATION MARKS, PARENTHESIS, RAY, SUBSCRIPT, SUPERSCRIPT, UPPER ARROWHEAD)		
LEFT ARROWHEAD		
barbed full —	XVII	87
barbed lower —	XVII	87
barbed upper —	XVII	87
blunted full —	XVII	87
blunted lower —	XVII	87
blunted upper —	XVII	87
curved full —	XVII	87
curved lower —	XVII	87
curved upper —	XVII	87
— in six-step rule for construction of arrows	137d	89
straight full —	XVII	87
straight lower —	XVII	87
straight upper —	XVII	87
LEFT-POINTING ARROW	XVI	75
LEFT-POINTING CARET	IX	44
	90c	51
— over equals sign	XVI	76
	132	86
LEFT PARENTHESIS ON COUNTERCLOCKWISE ARROW SHAFT	XVI	81
LENGTH OF ARROW SHAFT	139c	91
LESS THAN	XVI	75
	131k	85
— followed by equals sign followed by greater than	XVI	80
— followed by greater than	XVI	80
— with curved sides	XVI	75
(see also BAR, DISTINCTION BETWEEN, EQUALS SIGN, GREATER THAN, NEST OF)		
LETTER		
apostrophe-s combination with —s	15	11
capitalization of —s	45a	26
	46	26
	47a	27
	47c	27
	48b	27
— with ordinal ending	62	34
multipurpose indicator with —	96b	53
plural of —s	60	33
single enclosed —	26a	14
type form of —s	53	31
(see also CAPITALIZED LETTER, LETTER SEQUENCE, LOWER-CASE LETTERS, OTHER ALPHABETS, ROMAN LETTER, SINGLE LETTERS)		
LETTER SEQUENCE		
— of Roman letters	51b	29
— of Roman numerals	10c	8
LEVEL INDICATOR	VIII	34
	65	35
comma with —	73b	41
effectiveness of —	73	41
— with another —	73a	41
— with cancellation indicator	73c	42
— with more than three components	67d	37
— with one component	67a	35
— with punctuation indicator	73b	41
— with three components	67c	36
modified expression with —	83	48
orientation by —	67	35
space with —	73d	42
use of —s	66	35
(see also BASE-LINE INDICATOR, SUBSCRIPT INDICATOR, SUPERSCRIPT INDICATOR)		
LIMIT	XX	103
horizontal bar with —	88e	51
lower —	XX	103
	169	106
modifier with —	169	106
upper —	XX	103
	169	106
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE)		
LINE		
(see FULL LINE, HALF-LINE)		
LINEAR ARRANGEMENT WITHIN SPATIAL ARRANGEMENT		
— for cancellation	44a	24
— for fractions	39a	20
LINKED MATHEMATICAL EXPRESSIONS	184d	122
LITERARY ABBREVIATIONS	165	105
period after —	165	105
LITERARY BRACES	24	13
	147	96
non-use of —	26	14
use of —	25	13
LITERARY BRACKETS	24	13
	147	96
non-use of —	26	14
use of —	25	13
LITERARY COMMA	18	12
LITERARY ELLIPSIS	20	13
LITERARY OMISSION	42b	23
LITERARY PARENTHESES	24	13
	147	96
non-use of —	26	14
use of —	25	13
LOGARITHM	XX	103
	167	106
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE, NATURAL LOGARITHM)		
LOGICAL PRODUCT	XV	69
	121	73
	133e	87
(see also BAR, BAR OVER AND BAR UNDER, BAR OVER AND EQUALS SIGN UNDER, EQUALS SIGN, EQUALS SIGN OVER AND BAR UNDER, EQUALS SIGN OVER AND EQUALS SIGN UNDER)		
LOGICAL SUM	XV	69
	121	73
	133e	87
(see also BAR, BAR OVER AND BAR UNDER, BAR OVER AND EQUALS SIGN UNDER, EQUALS SIGN, EQUALS SIGN OVER AND BAR UNDER, EQUALS SIGN OVER AND EQUALS SIGN UNDER)		
LONG ARROW		
(see ARROW EAST, LONG ARROW SHAFT)		
LONG ARROW SHAFT		
double —	XVII	87
single —	XVII	87
LONG DASH	III	8
	19	13
— in omissions	19	13
	42b	23
punctuation of the —	19	13
LONG NUMERALS	6	7
comma with —	6	7
hyphen with —	6	7
runover of —	6	7
LOWER ARROWHEAD		
barbed left —	XVII	87
barbed right —	XVII	87
blunted left —	XVII	87
blunted right —	XVII	87
curved left —	XVII	87
curved right —	XVII	87
straight left —	XVII	87
straight right —	XVII	87
LOWER BARB		
(see ARROW EAST)		
LOWER-CASE EPSILON		
(see DISTINCTION BETWEEN)		
LOWER-CASE LETTERS	46	26
— in other alphabets	48b	27
— in Roman numerals	10a	8
	10b	8
LOWER INTEGRAL	XIX	98
LOWER LIMIT	XX	103
	169	106
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE)		
MACRON		
(see HORIZONTAL BAR)		
MAKES NEARER ARROWHEAD POINT NORTH	XVII	87
	138b	90
MAKES NEARER ARROWHEAD POINT SOUTH	XVII	87
	138b	90
MAPS INTO	131c	83
MATHEMATICAL BRACES	24	13
	147	96
use of —	148	96
(see also MATHEMATICAL GROUPING SYMBOLS)		
MATHEMATICAL BRACKETS	24	13
	147	96
use of —	148	96
(see also MATHEMATICAL GROUPING SYMBOLS)		
MATHEMATICAL COMMA	18	12
numeric indicator with —	3c	2
spacing with —	18	12
MATHEMATICAL ELLIPSIS	20	13
punctuation of —	20	13
MATHEMATICAL EXPRESSIONS		
apostrophe-s combination with —	15	11
contractions for to, into, by before —	108a	67
omission of —	41	23
plural of —	15	11
	76	44
references to —	184e	121
runover of linked —	184d	122
runover of —	184	121

Section Page
systems of — XII 56
98 57
(see also DETERMINANT, MATRIX)
MATHEMATICAL GROUPING SYMBOLS 147 96
use of — 148 96
(see also GROUPING SYMBOLS, ENLARGED GROUPING SYMBOLS)
MATHEMATICAL PARENTHESES 24 13
147 96
use of — 148 96
(see also MATHEMATICAL GROUPING SYMBOLS)
MATHEMATICAL SIGNIFICANCE 59a 33
type form with — 59b 33
type form without — 59a 33
MATRIX
— as system of mathematical expressions 98 57
numeric indicator with —s 4b 3
MAXIMUM XX 103
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE)
MEET XV 69
MEMBERSHIP XVI 75
131l 85
(see also DISTINCTION BETWEEN, REVERSE MEMBERSHIP)
MINIMUM XX 103
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE)
MINUS XV 69
122 73
— followed by plus XV 69
123 73
— or plus XV 69
124 73
numeric indicator with — symbol 3e 3
spacing with — symbol 175a 113
(see also CIRCLE WITH INTERIOR, DISTINCTION BETWEEN, PLUS)
MINUTES 160 101
164c 105
MIXED NUMBER 32 18
definition of — 32a 18
diagonal line in —s 32a 18
horizontal fraction line in —s 31a 17
32a 18
(see also FRACTIONAL PART OF MIXED NUMBER INDICATOR)
MODIFICATION INDICATOR IX 44
— in five-step rule for transcribing modified expressions 78c 46
(see also DIRECTLY-OVER INDICATOR, DIRECTLY-UNDER INDICATOR)
MODIFIED COMPARISON SIGN XVI 76
131g 84
131r 86
132 86
MODIFIED EXPRESSION
five-step rule for transcribing — 78 45
level indicator with — 83 48
— in five-step rule for transcribing —s 78b 46
— in subscripts and superscripts 83 48
multipurpose indicator with — 96a 53
plural of — 84 48
MODIFIER IX 44
arrow as — 87 49
definition of — 77 45
— of higher order 79 46
— of second order 79a 46
— of third order 79a 46
— with limit 169 106
Roman numerals which carry a — 10b 8
simultaneous —s 80 47
(see also INTERIOR MODIFIERS, WORD OR PHRASE IN THE ROLE OF A MATHEMATICAL EXPRESSION)

Section Page
MODULO XX 103
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE)
MULTIPLICATION XV 69
— cross XV 69
125a 74
— dot XV 69
125b 74
separation line in — 171a 110
special arrangement for — 174 113
MULTIPURPOSE INDICATOR X 53
comparison symbols with — 96c 54
decimal point with — 96d 54
interior modifiers with — 103b 65
letter with — 96b 53
modified expression with — 96a 53
— in five-step rule for transcribing modified expressions 78a 45
numeral with — 96b 53
remainder with — 170b 108
shape-modification symbols with — 96e 54
use of — 96 53
(see also DISTINCTION BETWEEN)
NABLA XIX 98
NATURAL LOGARITHM XX 103
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE)
NATURE OF SUBSCRIPTS AND SUPERSCRIPTS 63 35
NEGATION SIGN XVI 75
130 82
oblique —s XVI 75
vertical — XVI 75
(see also DISTINCTION BETWEEN)
NEGATION STROKE
(see NEGATION SIGN)
NEST OF
— greater than signs with curved sides XVI 81
135a 87
— greater than signs with straight sides XVI 81
135a 87
— less than signs with curved sides XVI 81
135a 87
— less than signs with straight sides XVI 81
135a 87
— radicals 97 54
NON-USE OF
— base-line indicator 72 41
78e 46
— capitalization indicator 46 26
— complex-fraction indicators 35 19
— contractions and short-form words 108 67
— enlargement indicator 99 58
— literary braces 26 14
— literary brackets 26 14
— literary parentheses 26 14
— numeric indicator 4 3
— omission indicator 42 23
— punctuation indicator 13 10
— Roman-letter indicator 52 30
— simple-fraction indicators 31 17
— subscript indicator 70 39
— type-form indicators 55 31
59 33
NORM 149a 97
NORMAL DIVISION FORMAT
(see DIVISION FORMAT)
NORMAL SYNTHETIC DIVISION FORMAT
(see SYNTHETIC DIVISION FORMAT)
NORTH
(see ARROW NORTH, ARROW SOUTH, BOLDFACE ARROW, CIRCLE WITH INTERIOR ARROWS, MAKES NEARER ARROWHEAD POINT NORTH)

Section Page
NORTHEAST
(see CORNER, SQUARE WITH INTERIOR)
NORTHWEST
(see CORNER, SQUARE WITH INTERIOR)
NULL SET 155 100
NUMBER SIGN XV 69
126 74
NUMERAL
apostrophe-s combination with —s 15 11
English Braille —s 2 1
long — 6 7
multipurpose indicator with —s 96b 53
170b 108
—s in figures and diagrams 9 7
—s which are not subscripts 96b 53
— with ordinal ending 7 7
numeric indicator with —s 3 1
4 3
58 32
plural of —s 8 7
representation of —s 2 1
single enclosed —s 26a 14
type forms of —s 57 32
(see also ENCLOSED NUMERALS, GROUPING SYMBOLS, ROMAN NUMERALS)
NUMERIC INDICATOR II 1
abbreviated scientific word or phrase with — 4a 3
argument with — 181 118
computation with — 4d 4
contractions for to, into, by with — 108a 67
dash with — 3 1
4 3
decimal point with — 3 1
4 3
determinant with — 4b 3
effectiveness of — 5 7
ellipsis with — 4c 4
enclosed numerals with — 148a 96
fraction with — 4d 4
hyphen with — 3c 2
mathematical comma with — 3c 2
matrix with — 4b 3
minus symbol with — 3e 3
non-use of — 4 3
numeral with — 3 1
4 3
— at beginning of braille line 3a 1
— in tables and charts 4d 4
punctuation associated with — 3c 2
regular polygon with — 58a 32
space with — 3b 2
4 3
spatial arrangement for fractions with — 4d 4
tables and charts with — 4d 4
type-form indicator with — 3d 2
57a 32
use of — 3 1
XXI 107
NUMERIC SUBSCRIPT 70 39
OBLIQUE ARROW DIRECTIONS 138c 90
OBLIQUE NEGATION SIGN XVI 75
OBTUSE ANGLE XIII 58
OBTUSE TRIANGLE XIII 58
OF THE FORM OF 133d 87
OMISSION V 23
ellipsis as — sign 41 23
hyphen as — sign 41 23
long dash in —s 19 13
space as — sign 41 23
(see also LITERARY OMISSION, OMISSION INDICATOR)
OMISSION INDICATOR V 23
non-use of — 42 23
number of —s 41 23
punctuation of — 43 23
spacing with — 43 23
use of — 41 23

Section Page
ONE-TO-ONE MAPPING 131c 83
OPEN INTERVAL 88a 50
OPENING
(see CANCELLATION INDICATOR, COMPLEX-FRACTION INDICATOR, FRACTIONAL-PART-OF-A-MIXED-NUMBER INDICATOR, HYPERCOMPLEX-FRACTION INDICATOR, SIMPLE-FRACTION INDICATOR)
OPERATION SIGNS XV 69
OPERATION SYMBOLS
boldface-type indicator with — 58a 32
spacing with — 175 113
OR XV 69
ORDER OF RADICAL 97a 54
97e 55
(see also ORDER-OF-RADICAL INDICATOR)
ORDER-OF-RADICAL INDICATOR XI 54
— in six-step rule for transcribing radicals 97a 54
97e 55
ORDER OF SUBSCRIPTS AND SUPERSCRIPTS 64 35
ORDINAL ENDINGS
contractions in — 7 7
108b 68
letters with — 62 34
numerals with — 7 7
punctuation of — 7 7
ORDINARY ARROW
— type XVII 87
140 92
(see also ARROW EAST, ARROW NORTH, ARROW SOUTH, ARROW WEST, BOLDFACE ARROW, ORDINARY ARROW SHAFT)
ORDINARY ARROW SHAFT
double — XVII 87
single — XVII 87
ORDINARY CARET
(see CARET)
ORDINARY EQUALS SIGN
(see EQUALS SIGN)
ORDINARY GREATER THAN
(see GREATER THAN)
ORDINARY INCLUSION
(see INCLUSION)
ORDINARY LESS THAN
(see LESS THAN)
ORDINARY MEMBERSHIP
(see MEMBERSHIP)
ORDINARY PIPESTRAP
(see PIPESTRAP)
ORDINARY SEPARATION LINE
(see SEPARATION LINE)
ORIENTATION BY LEVEL INDICATOR 67 35
OTHER ABBREVIATED SCIENTIFIC WORDS AND PHRASES 166 106
OTHER ALPHABETS 48b 27
drawing of letters from — 48b 27
lower-case letters in — 48b 27
shape indicator with — 48b 27
transcriber's note concerning — 48b 27
OTHER SHAPES 101 63
transcriber's note concerning — 100 62
102 64
OTHER SIGNS III 8
XIX 98
OUTER QUOTATION MARKS
left — III 8
right — III 8
OVAL XIII 58
OVER XV 69
(see also FRACTION LINE)
PAGE NUMBERS
English Braille — 2 1
references to — 2 1
184e 123

Section Page
PARAGRAPH MARK III 8
23 13
XV 69
110 71
— in references 23 13
110 71
PARALLELOGRAM XIII 58
PARENTHESIS
interlocking —s XVI 81
135b 87
left — III 8
XVIII 94
left — on counterclockwise arrow shaft XVI 81
literary —s 24 13
mathematical —s 147 96
right — III 8
XVIII 94
(see also ENLARGED PARENTHESIS, HORIZONTAL GROUPING SIGNS, USE OF LITERARY GROUPING SYMBOLS, USE OF MATHEMATICAL GROUPING SYMBOLS)
PARTIAL DERIVATIVE XIX 98
161 102
PENTAGON XIII 58
irregular — XIII 58
regular — XIII 58
PERCENT SIGN XIX 98
152 99
PERIOD III 8
27 14
— after abbreviations of measurement 27 14
164a 104
— after literary abbreviations 165 105
PERMISSIVE USE
— of spatial arrangement for fractions 40 22
PERSPECTIVE CORRESPONDENCE XVI 76
PHI
(see DISTINCTION BETWEEN)
PIPESTRAP XVI 75
131n 85
(see also REVERSE PIPESTRAP)
PLANCK'S CONSTANT 153 100
PLURAL
— of letters 15 11
60 33
— of mathematical expressions 15 11
76 44
— of modified expressions 84 48
— of numerals 8 7
— of shape signs 106 67
PLUS XV 69
127 74
— followed by minus XV 69
128 74
— or minus XV 69
129 74
spacing with — symbol 175a 113
(see also CIRCLE WITH INTERIOR, MINUS)
POLYGON
irregular — 101h 64
regular — 101h 64
POUND STERLING SIGN XIX 98
152 99
PRESCRIPTION XIX 98
PRIME XIX 98
160 101
—s in addition to subscripts and superscripts 75 43
Roman numerals which carry a — 10b 8
PRIORITY FOR RUNOVERS 184f 123
PROJECTIVE CORRESPONDENCE XVI 76
132 86
PROPORTION XVI 75
131p 86

Section Page
PUNCTUATION III 8
(see also PUNCTUATION ASSOCIATED WITH)
PUNCTUATION ASSOCIATED WITH
— apostrophe-s combination 15 11
— comparison symbol 178c 117
— long dash 19 13
— mathematical ellipsis 20 13
— numeric indicator 3c 2
— omission indicator 43 23
— ordinal endings 7 7
— Roman numerals 10d 8
— shape-modification symbols 178c 116
— shape symbols 178c 117
— single letters 61 33
PUNCTUATION INDICATOR III 8
11 10
comma with — 13a 10
dash with — 13a 10
effectiveness of — 14 11
hyphen with — 13a 10
non-use of — 13 10
— at beginning of braille line 13b 11
— with level indicator 73b 41
— with punctuation symbol 13b 11
space with — 13b 11
use of — 12 10
QUADRILATERAL XIII 58
QUANTIFIER
existential — XIX 98
156 100
universal — XIX 98
163 102
QUESTION MARK III 8
IX 44
94 53
— over equals sign 94 53
XVI 76
132 86
— under equals sign 94 53
QUOTATION MARKS III 8
single Roman letter within — 51c 30
(see also INNER QUOTATION MARKS, OUTER QUOTATION MARKS)
RADICAL XI 54
97c 55
— in six-step rule for transcribing —s 97c 55
six-step rule for transcribing —s 97 54
(see also INDEX OF RADICAL, INDEX-OF-RADICAL INDICATOR, LARGE RADICAL, NEST OF, ORDER OF RADICAL, ORDER-OF-RADICAL INDICATOR, RADICAL INDICATOR)
RADICAL INDICATOR XI 54
index-of- — XI 54
order-of- — XI 54
RADICAND
definition of — 97d 55
— in six-step rule for transcribing radicals 97d 55
RATIO XVI 75
131q 86
RAY
— extending to the left 87c 49
— extending to the right 87g 50
RECTANGLE XIII 58
(see also INTEGRAL WITH SUPERPOSED)
RECURRING DECIMAL
dot with — 91a 52
horizontal bar with — 88c 50
REFERENCES
asterisk in — 16 12
110 71
dagger in — 16 12
paragraph mark in — 23 13
110 71
— to numbered or lettered formulas 184e 123

Section Page

— to page numbers 2 1
184e 123
— to volume numbers 2 1
section mark in — 23 13
110 71
transcriber's note concerning transposition of — to numbered or lettered formula 184e 123
REGULAR HEXAGON XIII 58
REGULAR PENTAGON XIII 58
REGULAR POLYGON 101h 64
numeric indicator with — 58a 32
— of more than six sides 101h 64
REMAINDER
multipurpose indicator with — 170b 108
— in division format 170b 108
REPRESENTATION OF
— arrows XVII 87
— comparison signs compounded by superposition 135 87
— comparison signs compounded horizontally 134 87
— comparison signs compounded vertically 133 86
— corners 105a 66
— filled-in shapes 101a 63
— irregular polygons 101h 64
— numerals 2 1
— other alphabets 48b 27
— regular polygons 101h 64
— Roman numerals 10 8
— shapes 100 62
— shapes modified by superposition 104a 66
— shapes with interior modification 103a 65
— shapes with structural modification 102 64
REVERSE INCLUSION XVI 75
131j 85
(see also BAR, DOT, EQUALS SIGN, HORIZONTAL BAR, INCLUSION)
REVERSE MEMBERSHIP XVI 75
131m 85
(see also MEMBERSHIP)
REVERSE PIPESTRAP XVI 75
131o 85
(see also PIPESTRAP)
REVERSE SYNTHETIC DIVISION FORMAT XXI 107
(see also SYNTHETIC DIVISION FORMAT)
REVERSIBLE REACTION 131c 83
RHOMBUS XIII 58
RIGHT
(see ANGLE BRACKET, BARBED ARROWHEAD, BARRED BRACE, BARRED BRACKET, BLUNTED ARROWHEAD, BOLDFACE BRACKET, BRACE, BRACKET, CURVED ARROWHEAD, ENLARGED BRACE, ENLARGED BRACKET, ENLARGED PARENTHESIS, FULL ARROWHEAD, INNER QUOTATION MARKS, LOWER ARROWHEAD, OUTER QUOTATION MARKS, PARENTHESIS, RAY, RIGHT ANGLE, RIGHT ARROWHEAD, RIGHT-POINTING ARROW, RIGHT-POINTING CARET, RIGHT TRIANGLE, SUBSCRIPT, SUPERSCRIPT, UPPER ARROWHEAD)
RIGHT ANGLE XIII 58
RIGHT ARROWHEAD
barbed full — XVII 87
barbed lower — XVII 87
barbed upper — XVII 87
blunted full — XVII 87
blunted lower — XVII 87
blunted upper — XVII 87
curved full — XVII 87
curved lower — XVII 87

Section Page

curved upper — XVII 87
— in six-step rule for construction of arrows 137f 89
straight full — XVII 87
straight lower — XVII 87
straight upper — XVII 87
RIGHT-POINTING ARROW XVI 75
RIGHT-POINTING CARET IX 44
90d 51
— over equals sign XVI 76
132 86
RIGHT TRIANGLE XIII 58
ROMAN ALPHABET 48a 27
APPENDIX 128
ROMAN-LETTER INDICATOR VII 26
contraction for to, into, by with — 108a 67
non-use of — 52 30
— with Roman letters 51 29
— with Roman numerals 10a 8
10b 8
use of — 51 29
ROMAN LETTERS
Roman-letter indicator with — 51 29
— in computation 52c 30
— in formulas 52c 30
sequence of — 52b 30
(see also ROMAN-LETTER INDICATOR)
ROMAN NUMERALS 10 8
capitalized — 10a 8
10b 8
10c 8
45b 26
English Braille — 10a 8
10d 8
lower-case — 10a 8
10b 8
mathematical grouping symbols with — 148a 96
punctuation of — 10d 8
Roman-letter indicator with — 10a 8
10b 8
— in computation 10b 8
— in formulas 10b 8
— of one letter 10a 8
— of two or more letters 10a 8
— which carry a modifier 10b 8
— which carry a prime 10b 8
— which carry a subscript 10b 8
— which carry a superscript 10b 8
ROUND BRACKETS XVIII 94
ROUND D XIX 98
161 102
RUNOVER XXIII 120
priority for —s 184f 123
— in exercises 184c 121
— in linked mathematical expressions 184d 122
— in long numerals 6 7
— in mathematical expressions 184 121
— in spatial arrangement for fractions 39a 20
RUSSIAN ALPHABET 48a 27
APPENDIX 129
RUSSIAN-LETTER INDICATOR VII 26
SANSERIF-TYPE INDICATOR VII 26
54 31
SCALENE TRIANGLE XIII 58
SCRIPT-TYPE INDICATOR VII 26
SECANT XX 103
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE, HYPERBOLIC)
SECONDS 160 101
164c 105
SECTION MARK III 8
23 13
XV 69
110 71
— in references 23 13

Section Page

SEMICOLON III 8
SEPARATION LINE XXI 107
171 109
— in addition 171a 110
— in carrying and borrowing XXI 107
171b 111
— in changing number forms XXI 107
— in division 171c 111
— in division format 170a 108
— in extraction of a root 171c 111
— in multiplication 171a 110
— in subtraction 171a 110
spacing with — 183 120
SHAPE INDICATOR XIII 58
capitalization indicator with — 47b 27
— in six-step rule for construction of arrows 137a 89
— with other alphabets 48b 27
— with shape symbols 100 62
SHAPE-MODIFICATION INDICATOR XIII 58
— with interior modifiers 103b 65
103c 65
— with shape-modification symbols 102 64
103a 65
SHAPE-MODIFICATION SYMBOL
braille indicator with — 178a 116
multipurpose indicator with — 96e 54
punctuation with — 178c 117
shape-modification indicator with — 102 64
—s associated to form a comparison symbol compounded vertically 178b 117
spacing with — 178 116
SHAPE MODIFIED BY SUPERPOSITION XIII 58
104 65
definition of — 104 65
representation of — 104a 66
104b 66
SHAPE SIGNS
basic — XIII 58
definition of — 100 62
drawing of — 100 62
other — 101 63
plural of — 106 67
representation of — 100 62
(see also SHAPE MODIFIED BY SUPERPOSITION, SHAPE WITH INTERIOR MODIFICATION, SHAPE WITH STRUCTURAL MODIFICATION)
SHAPE SYMBOLS
boldface-type indicator with — 58a 32
101a 63
drawing of — 100 62
punctuation of — 178c 117
shape indicator with — 100 62
spacing with — 175b 114
178 116
SHAPE WITH INTERIOR MODIFICATION XIII 58
103 65
definition of — 103a 65
representation of — 103a 65
SHAPE WITH STRUCTURAL MODIFICATION XIII 58
102 64
definition of — 102 64
representation of — 102 64
SHORT ARROW
(see ARROW EAST, SHORT ARROW SHAFT)
SHORT ARROW SHAFT
double — XVII 87
single — XVII 87
SHORT DASH III 8
numeric indicator with — 3 1
4 3
punctuation indicator with — 13a 10

Section Page
SHORT-FORM WORDS
  (see CONTRACTIONS)
SIGNS
  (see BOLDFACE COMPARISON SIGNS, BOLDFACE EQUALS SIGN, CENT SIGN, COMPARISON SIGNS, DEGREE SIGN, DOLLAR SIGN, ENLARGED GROUPING SIGNS, EQUALS SIGN, EQUALS SIGN OVER AND BAR UNDER, EQUALS SIGN OVER AND EQUALS SIGN UNDER, FORMAT SIGNS, GROUPING SIGNS, HORIZONTAL GROUPING SIGNS, MODIFIED COMPARISON SIGN, NEGATION SIGN, NUMBER SIGN, OPERATION SIGNS, OTHER SIGNS, PERCENT SIGN, POUND STERLING SIGN, SHAPE SIGNS, SIMPLE COMPARISON SIGNS)
SIMPLE COMPARISON SIGNS XVI 75
  131 83
  (see also HORIZONTAL BAR, TILDE)
SIMPLE FRACTION 29 16
  definition of — 29 16
  (see also SIMPLE-FRACTION INDICATOR, SIMPLE-FRACTION LINE)
SIMPLE-FRACTION INDICATOR IV 15
  closing — IV 15
  non-use of —s 31 17
  opening — IV 15
  —s with complex fraction 30c 17
  use of — 30 16
SIMPLE-FRACTION LINE IV 15
  XV 69
SIMPLE TILDE IX 44
  95 53
  XVI 75
  131t 86
  (see also DOT)
SIMULTANEOUS MODIFIERS 80 47
SIMULTANEOUS SUBSCRIPTS AND SUPERSCRIPTS 74 43
SINCE XIX 98
  162 102
SINE XX 103
  (see also ABBREVIATED SCIENTIFIC WORD OR PHRASE, HYPERBOLIC)
SINGLE ARROW SHAFT
  long — XVII 87
  ordinary — XVII 87
  short — XVII 87
SINGLE BOLDFACE VERTICAL BAR XVIII 94
SINGLE ENCLOSED LETTER 26a 14
SINGLE ENCLOSED NUMERAL 26a 14
SINGLE INTEGRAL XIX 98
SINGLE LETTER
  mathematical grouping symbols with — 148a 96
  punctuation of — 61 33
  (see also SINGLE ROMAN LETTER)
SINGLE ROMAN LETTER
  — as Roman numeral 10a 8
  — with abbreviations of measurement 51d 30
  — within quotation marks 51c 30
SINGLE TILDE
  (see BAR, EQUALS SIGN, SIMPLE TILDE, TILDE)
SINGLE VERTICAL BAR XVIII 94
  (see also BOLDFACE VERTICAL BAR, ENLARGED VERTICAL BAR)
SIX-STEP RULE FOR CONSTRUCTION OF ARROWS 137 89
  arrow direction in — 137b 89
  arrow shaft in — 137e 89
  arrow type in — 137c 89

Section Page
  left arrowhead in — 137d 89
  right arrowhead in — 137f 89
  shape indicator in — 137a 89
SIX-STEP RULE FOR TRANSCRIBING RADICALS 97 54
  index of radical in — 97b 54
  index-of-radical indicator in — 97b 54
  order-of-radical indicator in — 97a 54
  97e 55
  radical in — 97c 55
  radicand in — 97d 55
  termination indicator in — 97f 55
SMALL CIRCLE XIII 58
SOUTH
  (see ARROW NORTH, ARROW SOUTH, BOLDFACE ARROW, CIRCLE WITH INTERIOR ARROWS, MAKES NEARER ARROWHEAD POINT SOUTH)
SOUTHEAST
  (see CORNER, SQUARE WITH INTERIOR)
SOUTHWEST
  (see CORNER, SQUARE WITH INTERIOR)
SPACE
  level indicator with — 73d 42
  73e 42
  73f 43
  numeric indicator with — 3b 2
  4 3
  punctuation indicator with — 13b 11
SPACING XXII 113
  — rule for argument 182 119
  — to achieve alignment 175d 114
  — with abbreviated scientific word or phrase 180 117
  — with abbreviations of measurement 179 117
  — with check mark 17 12
  — with comparison symbols 176 115
  — with ditto marks 17 12
  — with grouping symbols 177 116
  — with mathematical comma 18 12
  — with minus symbol 175a 113
  — with omission indicator 43 23
  — with operation symbols 175 113
  — with plus symbol 175a 113
  — with separation line 183 120
  — with shape-modification symbols 178 116
  — with shape symbols 175b 114
  178 116
  — with spatial arrangements 185 123
  — with tally marks 28 14
  — with word or phrase in the role of a mathematical expression 175c 114
SPATIAL ARRANGEMENT XXIII 120
  spacing with —s 185 123
  (see also LINEAR ARRANGEMENT WITHIN SPATIAL ARRANGEMENT, SPATIAL ARRANGEMENT FOR CANCELLATION, SPATIAL ARRANGEMENT FOR FRACTIONS)
SPATIAL ARRANGEMENT FOR CANCELLATION 44a 24
  linear arrangement within — 44a 24
SPATIAL ARRANGEMENT FOR FRACTIONS 39 20
  linear arrangement within — 39a 20
  numeric indicator with — 4d 4
  permissive use of — 40 22
  — in continued fractions 38 20
  39b 21
  — in hypercomplex fractions 37 19
  use of — 39 20
SPEAR 139b 91
  — west-east followed by identity sign followed by tilde XVI 80

Section Page
SPECIAL ARRANGEMENT FOR MULTIPLICATION 174 113
SQUARE XIII 58
  filled-in — XV 69
  116 72
  hollow — XV 69
  119 72
  (see also INTEGRAL WITH SUPERPOSED, SQUARE WITH INTERIOR, SQUARE WITH SUPERPOSED)
SQUARE BRACKETS XVIII 94
SQUARE ROOT XI 54
  separation line in extracting a — 171c 111
  (see also SQUARE ROOT FORMAT)
SQUARE ROOT FORMAT XXI 107
  172 112
SQUARE WITH INTERIOR
  — diagonals XIII 58
  — dot XIII 58
  — horizontal bar XIII 58
  — northwest-southeast diagonal XIII 58
  — southwest-northeast diagonal XIII 58
  — vertical bar XIII 58
SQUARE WITH SUPERPOSED
  — horizontal bar XIII 58
  — vertical bar XIII 58
STAR
  (see ASTERISK)
STRAIGHT ANGLE XIII 58
STRAIGHT ARROWHEAD
  left full — XVII 87
  left lower — XVII 87
  left upper — XVII 87
  right full — XVII 87
  right lower — XVII 87
  right upper — XVII 87
STRAIGHT SIDES
  (see DIVISION FORMAT, NEST OF)
STRUCTURAL MODIFICATION
  (see SHAPE WITH STRUCTURAL MODIFICATION)
SUBDIVISIONS IN EXERCISES 184c 121
SUBSCRIPT VIII 34
  comma in —s 73b 41
  definition of — 63 35
  left — 68 38
  numerals which are not —s 96b 53
  numeric — 70 39
  right — 68 38
  Roman numerals which carry a — 10b 8
  — containing a modified expression 83 48
  — of first order 64 35
  — of higher order 64 35
  — of second order 64 35
  — of third order 64 35
  (see also SUBSCRIPT INDICATOR, SUBSCRIPTS AND SUPERSCRIPTS, WHOLE-WORD MEASUREMENTS, WORD OR PHRASE IN THE ROLE OF A MATHEMATICAL EXPRESSION)
SUBSCRIPT INDICATOR VIII 34
  non-use of — 70 39
  — with arrow directions 138c 90
SUBSCRIPTS AND SUPERSCRIPTS
  direct — 69 39
  nature of — 63 35
  order of — 64 35
  primes in addition to — 75 43
  simultaneous — 74 43
SUBSCRIPT TO SUBSCRIPT VIII 34
SUBSCRIPT TO SUBSCRIPT TO SUBSCRIPT VIII 34
SUBSCRIPT TO SUBSCRIPT TO SUPERSCRIPT VIII 34
SUBSCRIPT TO SUPERSCRIPT VIII 34
SUBSCRIPT TO SUPERSCRIPT TO SUBSCRIPT VIII 34
SUBSCRIPT TO SUPERSCRIPT TO SUPERSCRIPT VIII 34

Section Page
SUBTRACTION
separation line in — 171a 110
SUCH THAT XIX 98
SUPERPOSED ARC
(see ANGLE)
SUPERPOSITION
(see COMPARISON SIGNS, DISTINCTION BETWEEN, SHAPE MODIFIED BY SUPERPOSITION, SUPERPOSITION INDICATOR)
SUPERPOSITION INDICATOR XIII 58
— with superposed symbols 104 66
SUPERSCRIPT VIII 34
asterisk in — position 16 12
dagger in — position 16 12
definition of — 63 35
hollow dot as — 118 72
left — 68 38
right — 68 38
Roman numerals which carry a — 10b 8
— containing a modified expression 83 48
— of first order 64 35
— of higher order 64 35
— of second order 64 35
— of third order 64 35
(see also SUBSCRIPTS AND SUPERSCRIPTS, SUPERSCRIPT INDICATOR, WHOLE-WORD MEASUREMENTS, WORD OR PHRASE IN THE ROLE OF A MATHEMATICAL EXPRESSION)
SUPERSCRIPT INDICATOR VIII 34
— with arrow directions 138c 90
SUPERSCRIPT TO SUBSCRIPT VIII 34
SUPERSCRIPT TO SUBSCRIPT TO SUBSCRIPT VIII 34
SUPERSCRIPT TO SUBSCRIPT TO SUPERSCRIPT VIII 34
SUPERSCRIPT TO SUPERSCRIPT VIII 34
SUPERSCRIPT TO SUPERSCRIPT TO SUBSCRIPT VIII 34
SUPERSCRIPT TO SUPERSCRIPT TO SUPERSCRIPT VIII 34
SUPPLEMENTARY ANGLES XIII 58
SUPREMUM XX 103
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE)
SYMBOLS
(see COMPARISON SYMBOLS, ENLARGED GROUPING SYMBOLS, FORMAT SYMBOLS, GROUPING SYMBOLS, MATHEMATICAL GROUPING SYMBOLS, OPERATION SYMBOLS, SHAPE-MODIFICATION SYMBOLS, SHAPE SYMBOLS)
SYNTHETIC DIVISION FORMAT XXI 107
173 112
normal — XXI 107
reverse — XXI 107
SYSTEM OF MATHEMATICAL EXPRESSIONS 98 57
determinant as — 98 57
matrix as — 98 57
TABLES AND CHARTS
numeric indicator with — 4d 4
TALLY III 8
28 14
spacing with — marks 28 14
TANGENT XX 103
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE, HYPERBOLIC)
TERMINATING THE ARGUMENT
criteria for — 181 118

Section Page
TERMINATION INDICATOR XI 54
— in five-step rule for transcribing modified expressions 78e 46
— in six-step rule for transcribing radicals 97f 55
— with corners 105a 66
THERE EXISTS XIX 98
156 100
— uniquely 156 100
THEREFORE XIX 98
162 102
TIC-TAC-TOE
(see NUMBER SIGN)
TILDE IX 44
95 53
XVI 75
131r 86
— over simple comparison sign 131r 86
— under simple comparison sign 131r 86
(see also CARET, DOT, DOUBLE TILDE, EXTENDED TILDE, SIMPLE TILDE, SINGLE TILDE, SPEAR)
TIMES
(see MULTIPLICATION)
TO, INTO, BY
(see CONTRACTIONS FOR TO, INTO, BY)
TRANSCRIBER'S NOTE CONCERNING
— omission of arrows from vectors 58b 33
— other alphabets 48b 27
— other shape signs 101 63
102 64
— transposition of reference to numbered or lettered formula 184e 123
TRANSCRIBING MODIFIED EXPRESSIONS
(see FIVE-STEP RULE FOR TRANSCRIBING MODIFIED EXPRESSIONS)
TRANSCRIBING RADICALS
(see SIX-STEP RULE FOR TRANSCRIBING RADICALS)
TRAPEZOID XIII 58
TRIANGLE XIII 58
acute — XIII 58
equilateral — IX 44
92 52
XIII 58
isosceles — XIII 58
obtuse — XIII 58
right — XIII 58
scalene — XIII 58
TRIGONOMETRIC FUNCTION 168 106
TRIPLE BAR
(see DISTINCTION BETWEEN)
TRIPLE INTEGRAL XIX 98
TWO-WAY ARROW XVI 75
131c 83
TYPE FORM VII 26
— of letters 53 31
— of numerals 57 32
—with mathematical significance 59b 33
— without mathematical significance 59a 33
(see also TYPE-FORM INDICATOR)
TYPE-FORM INDICATOR VII 26
alphabetic indicator with — 54 31
effectiveness of — 56 32
non-use of — 55 31
59 33
numeric indicator with — 3d 2
57a 32
use of — 54 31

Section Page
(see also BOLDFACE-TYPE INDICATOR, ITALIC-TYPE INDICATOR, SANSERIF-TYPE INDICATOR, SCRIPT-TYPE INDICATOR)
UNCAPITALIZED LETTERS
(see LOWER-CASE LETTERS)
UNION XV 69
120 72
133b 87
(see also BAR, EQUALS SIGN)
UNIVERSAL QUANTIFIER XIX 98
163 102
UPPER ARROWHEAD
barbed left — XVII 87
barbed right — XVII 87
blunted left — XVII 87
blunted right — XVII 87
curved left — XVII 87
curved right — XVII 87
straight left — XVII 87
straight right — XVII 87
UPPER BARB
(see ARROW EAST)
UPPER INTEGRAL XIX 98
UPPER LIMIT XX 103
169 106
USE OF
— base-line indicator 71 40
— cancellation indicators 44 24
— capitalization indicator 45 26
— complex-fraction indicators 34 18
— contractions and short-form words 107 67
— enlargement indicator 98 57
— hypercomplex-fraction indicators 37 19
— level indicators 66 35
— literary braces 25 13
— literary brackets 25 13
— literary parentheses 25 13
— mathematical grouping symbols 148 96
— multipurpose indicator 96 53
— numeric indicator 3 1
XXI 107
— omission indicator 41 23
— punctuation indicator 12 10
— Roman-letter indicator 51 29
— simple-fraction indicators 30 16
— spatial arrangement for fractions 39 20
— type-form indicators 54 31
VARIATION XVI 75
131u 86
VARIES AS XVI 75
131u 86
VECTORS
arrow with — 58b 33
boldface type with — 58b 33
transcriber's note concerning omission of arrows from — 58b 33
VERSINE XX 103
(see also ABBREVIATED SCIENTIFIC WORD OR PHRASE)
VERTICAL ANGLES XIII 58
VERTICAL ARROW DIRECTIONS 138b 90
VERTICAL BAR IX 44
89 51
XVIII 94
149 97
XIX 98
— over equals sign XVI 76
(see also ARROW EAST, ARROW WEST, CIRCLE WITH INTERIOR, CIRCLE WITH SUPERPOSED, DOUBLE VERTICAL BAR, ENLARGED VERTICAL BAR, SINGLE VERTICAL BAR, SQUARE WITH INTERIOR, SQUARE WITH SUPERPOSED)

Section Page

VOID SET 155 100
VOLUME NUMBERS
  English Braille — 2 1
  references to — 2 1
WEIERSTRASS P XIX 98
WEST
  (see ARROW EAST, ARROW WEST, CIRCLE WITH INTERIOR ARROWS)
WHOLE-WORD MEASUREMENT 164b 105

Section Page

  — carrying a subscript or superscript 108f 68
WORD OR PHRASE IN THE ROLE OF A MATHEMATICAL EXPRESSION
  contractions in — 108f 68
  177 116
  definition of — 148b 97
  fraction containing — 39c 21
  grouping symbols with — 148b 97
  177 116

Section Page

  spacing with — 175c 114
  — carrying a modifier 108f 68
  — carrying a subscript or superscript 108f 68
  — which is a modifier 108f 68
  — which is a subscript or superscript 108f 68
WORK ARRANGED FOR COMPUTATION
  (see COMPUTATION)
YIELDS 131c 83

HV1698 Nemeth, Abraham.
N343 The Nemeth Code of c.4
1965 braille mathematics and
scientific notation 1965.

DATE DUE

AMERICAN FOUNDATION FOR THE BLIND
15 WEST 16th STREET
NEW YORK, N. Y. 10011